# A NOTE ON THIRD-DEGREE PRICE DISCRIMINATION IN OLIGOPOLY 

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#### Abstract

In this note, we use a two-stage game to examine the domination of uniform price by third-degree price discrimination in a $k$-firm, two-market model. Not surprisingly, when $k=1$, a monopoly case results. Accordingly, the price, output, profit, and weffare effects under third-degree price discrimination in an oligopoly with symmetric linear demands and constant marginal costs are the same as the results obtained in monopoly price discrimination.


Keywords: Third-degree price discrimination, Oligopoly, Differentiated product
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## 1. Introduction

Third-degree price discrimination occurs when a firm can charge different prices to consumers who have different characteristics. Traditionally the analysis is confined to monopoly environments. Varian (1989) and Dastidar (2006) provide succinct summaries of this literature. Recently papers such as Holmes (1989), Cheung and Wang (1997), Corts (1998), Azar (2003), Dastidar (2006), and Galera and Zaratiegui (2006) have analyzed the effects of third-degree price discrimination in an oligopolistic industry structure. However, except for Cheung and Wang (1997), they consider a duopolistic framework. Cheung and Wang (1997) discuss output effects under third-degree price


discrimination in a $n$-firm quantity-setting oligopoly. "However, it may be argued that price-setting oligopoly provides a better framework" (Dastidar, 2006). This note considers a $k$-firm, two-market model with linear demands and constant marginal costs to examine whether some firms will choose price discrimination and others will select uniform price in an oligopoly.

Corts (1998) argues that "competitive price discrimination may intensify competition by giving firms more weapons with which to wage their war." In an oligopoly, market competition is usually increased by a rise in the number of firms. To relax competition, firms may wish not to discriminate to increase profits. Furthermore, Holmes (1989) argues that "the firms in an oligopoly may be worse off with a larger choice set." Besides, Azar (2003) also mentions that "..., with oligopoly firms are sometimes better off being constrained because their rivals are also constrained and because of the strategic effects of the constrained." In other words, price discrimination does not guarantee that a firm can increase its profits. Are there a number of firms in each regime that would equate the profits of the two pricing policies, where no firm would gain by switching policies unilaterally? If so, some firms would practice price discrimination and others would not. Kutlu (2009) examines the effects of second-degree price discrimination in the Stackelberg model and shows that the leader practices no price discrimination whatsoever; rather, the follower does all the price discrimination.

In view of economic welfare, a well known necessary condition for (monopolistic or oligopolistic) welfare to increase under third-degree price discrimination is that total output should increase with discrimination. Although Adachi (2002) suggests that welfare may improve even if total output remains constant by incorporating interdependence into linear demands in monopoly environments, Bertoletti (2004) argues that, very generally, with price discrimination welfare must decrease when total output does not change. In recent years there have been papers such as Schulz (1999), Liu and Serfes (2010), Adachi and Matsushima (2011) and Cowan (2013) indicate that price discrimination can improve welfare. However, it would be dangerous to draw any such conclusion. Armstrong (2006) explains the economic motives for price discrimination and outlines the impacts of this practice on consumers, rivals and total welfare. He argues that "policy towards price discrimination should be founded on good economic understanding of the market in question." In other words, price discrimination does not necessarily turn out to be harmless from a social point of view.
We find that when firms simultaneously and independently select prices in the product markets, after an ex ante evaluation, they see that uniform price is dominated by price discrimination. Accordingly, using uniform price is not rational, because playing price discrimination, regardless of what the other players do in the game, can increase profits. The reason is the same as in the monopoly case: each firm is better off unconstrained in its price selection when all choice profiles of the other firms are given. In our model, firms simultaneously choose between uniform price and third-degree price discrimination is not essential, because of the domination of the uniform price by the price discrimination. In this symmetric model, if we let the number of firms be equal to one, it results in the monopoly case. Consequently, the price, output, profit, and welfare effects under third-degree price discrimination are the same in an oligopoly as the results in monopoly price discrimination.

This note is organized as follows. The next section of the note presents the model and results. Conclusions are contained in Section 3.



## 2. The Model and Results

There are $k$ firms that sell differentiated products and compete in prices; competition takes place over two markets, denoted by 1 and 2 . It is feasible for each firm to set different prices in the two markets and to prevent resale or consumer arbitrage. In other words, the firms can practice thirddegree price discrimination. Let $q_{i j}$ be the quantity demanded from firm $j$ by buyers in market $i$, and $p_{i j}$ be the price set by firm $j$. Note that $i=1,2$ and $j=1,2, \cdots, k$. There is a representative consumer with a quadratic utility function in market $i$ :

$$
\begin{equation*}
U_{i}=m_{i}+a_{i} \sum_{j=1}^{k} q_{i j}-\frac{1}{2} \sum_{j=1}^{k} q_{i j}^{2}-b \sum_{j=1}^{k} q_{i j} \sum_{f \neq j}^{k} q_{i f}, m_{i}, a_{i}, b>0 . \tag{1}
\end{equation*}
$$

Accordingly, the inverse demand for firm $j$ in market $i$ is given by $p_{i j}=a_{i}-q_{i j}-b \sum_{f \neq j}^{k} q_{i f}$, that is, the demand for firm $j$ in market $i$ is:

$$
\begin{equation*}
q_{i j}=\frac{(1-b) a_{i}-(1+(k-2) b) p_{i j}+b \sum_{f \neq j}^{k} p_{i f}}{(1-b)(1+(k-1) b)} . \tag{2}
\end{equation*}
$$

Firm $j$ has a constant marginal cost of production, $c_{j}$. Let $\pi_{i j}$ be the profits of firm $j$ in market $i$, that is:

$$
\begin{gather*}
\pi_{i j}=p_{i j} q_{i j}-c_{j} q_{i j}=\left(p_{i j}-c_{j}\right) \frac{(1-b) a_{i}-(1+(k-2) b) p_{i j}+b \sum_{f \neq j}^{k} p_{i f}}{(1-b)(1+(k-1) b)} \\
=P_{i j} \frac{(1-b) A_{i}-(1+(k-2) b) P_{i j}+b \sum_{f \neq j}^{k} P_{i f}}{(1-b)(1+(k-1) b)}=P_{i j} Q_{i j}, \tag{3}
\end{gather*}
$$

where $P_{i j}=\left(p_{i j}-c_{j}\right)$ and $A_{i}=a_{i}-\left[(1+(k-2) b) c_{j}-b \sum_{f \neq j}^{k} c_{f}\right] /(1-b)$.

Without loss of generality, from now on costs are normalized to zero, that is, $\pi_{i j}=p_{i j} q_{i j}$. The timing of events is as follows. In stage one, firms simultaneously and independently choose between uniform price and third-degree price discrimination. In stage two, firms simultaneously and

independently compete by selecting prices. To find the subgame perfect Nash equilibrium of the game, we proceed by solving the game backwards.

Stage two: Suppose that $m$ firms do not practice third-degree price discrimination, and $n$ firms practice third-degree price discrimination in the first stage, where $m+n=k$. Let $p_{r}\left(q_{r}\right)$ be the uniform price (quantity) of a nondiscrimination firm, where $r=1,2, \cdots, m$. Note that $p_{1 r}=p_{2 r}=p_{r}$ and $q_{r}=q_{1 r}+q_{2 r}$. Let firm $t$ be a discrimination firm, where $t=m+1, m+2, \cdots, m+n$. Accordingly, firm $r$ 's demand is as follows:

$$
\begin{equation*}
q_{r}=q_{1 r}+q_{2 r}=\frac{(1-b)\left(a_{1}+a_{2}\right)-2(1+(k-2) b) p_{r}+2 b \sum_{s \neq r}^{m} p_{s}+b \sum_{t=m+1}^{m+n}\left(p_{1 t}+p_{2 t}\right)}{(1-b)(1+(k-1) b)} . \tag{4}
\end{equation*}
$$

Now, simultaneously and independently, firm $r$ chooses $p_{r}$ to maximize $\pi_{r}=p_{r} q_{r}$, and firm $t$ chooses $p_{i t}$ to maximize $\pi_{i t}=p_{i t} q_{i t}$ in market $i$. The first order conditions are as follows:

$$
\begin{align*}
& \frac{d \pi_{r}}{d p_{r}}=q_{r}-\frac{2(1+(k-2) b)}{(1-b)(1+(k-1) b)} p_{r}=0,  \tag{5}\\
& \frac{d \pi_{i t}}{d p_{i t}}=q_{i t}-\frac{(1+(k-2) b)}{(1-b)(1+(k-1) b)} p_{i t}=0 . \tag{6}
\end{align*}
$$

Let $p^{n d}$ and $p_{i}^{d}$ be the equilibrium price for nondiscrimination firms, and the equilibrium price for discrimination firms in market $i$, respectively. The symmetry assumption implies that $p_{1}=p_{2}=\cdots=p_{m}=p^{n d}$, and $p_{i m+1}=p_{i m+2}=\cdots=p_{i m+n}=p_{i}^{d}$. Solving Eqs. (5) and (6) gives us

$$
\begin{gather*}
p^{n d}(m, n)=\frac{(1-b)\left(a_{1}+a_{2}\right)}{2(2+(k-3) b)},  \tag{7}\\
p_{i}^{d}(m, n)=\frac{(1-b)\left[2(2+(k-3) b) a_{i}+m b\left(a_{1}+a_{2}\right)\right]}{2(2+(k-3) b)(2+(2 k-n-3) b)} . \tag{8}
\end{gather*}
$$

Let $q^{n d}\left(\pi^{n d}\right)$ and $q_{i}^{d}\left(\pi_{i}^{d}\right)$ be the equilibrium quantity (profits) for nondiscrimination firms, and the equilibrium quantity (profits) for discrimination firms in market $i$, respectively. From Eqs. (5) and (6) we have $q^{n d}$ and $q_{i}^{d}$. Let $\pi^{d}$ be the profits for discrimination firms, that is $\pi^{d}=\pi_{1}^{d}+\pi_{2}^{d}$. Consequently,


$$
\begin{gather*}
\pi^{n d}=\frac{2(1+(k-2) b)}{(1-b)(1+(k-1) b)}\left(p^{n d}\right)^{2},  \tag{9}\\
\pi^{d}=\frac{(1+(k-2) b)}{(1-b)(1+(k-1) b)}\left(\left(p_{1}^{d}\right)^{2}+\left(p_{2}^{d}\right)^{2}\right) . \tag{10}
\end{gather*}
$$

It can be easily verified that $q^{n d}=q_{1}^{d}+q_{2}^{d}$ and $p^{n d}=\left(p_{1}^{d}+p_{2}^{d}\right) / 2$. This is the same as the results in monopoly environments with linear demands and constant marginal costs. Not surprisingly, when $k=m=1, n=0(k=n=1, m=0)$, it is the case that a monopoly practices uniform price (price discrimination).

Stage one: Now we can determine the number of firms in each regime that would be required to equate the profits of the two pricing policies, where no firm would gain by switching policies unilaterally. Consequently, some firms would practice price discrimination and others would not. However, a firm that was planning to practice uniform price would strictly gain by switching to the action of practicing price discrimination, that is, uniform price is dominated by price discrimination for all firms. Let us consider the case that one of the nondiscrimination firms alters its action and becomes a discrimination firm. This means that there will be $m-1$ nondiscrimination firms and $n+1$ discrimination firms. Accordingly, if the condition $\pi^{d}(m-1, n+1)>\pi^{n d}(m, n)$ is satisfied for $1 \leq m \leq k$ and $n=k-m$; that is, each nondiscrimination firm has the incentive to become a discrimination firm, then we have the following proposition:

Proposition 1. In equilibrium, all the firms will choose price discrimination.
Proof. It is easy to show that

$$
\begin{equation*}
\pi^{d}(m-1, n+1)-\pi^{n d}(m, n)=\frac{(1-b)(1+(k-2) b)\left(a_{1}-a_{2}\right)^{2}}{2(1+(k-1) b)(2+(k+m-4) b)^{2}}>0 . \tag{11}
\end{equation*}
$$

In simultaneous games, players form beliefs about others' behavior and consider this as expected values, so that each player is better off unconstrained in its selection, as in the monopoly environments. We suspect that proposition 1 is still robust, even if demand and cost functions are general, the number of markets is any positive integer, and firms simultaneously and independently compete by selecting quantities. Furthermore, even if firms sequentially select prices, price discrimination is still better off under complete information. The point is that we can find the subgame perfect Nash equilibrium by backward induction. Having observed the other firms' actions in the proceeding stages, firms are better off using price discrimination in the last stage as in simultaneous games. In the second last stage, firms are better off using price discrimination for the outcomes in the last stage are evaluated and the outcomes of the proceeding stages are given. Similarly, firms find it is better to practice price discrimination rather than uniform pricing in each stage. Now we will check whether firms will find themselves in a prisoner's dilemma.

Proposition 2. The profits of a firm are higher under price discrimination in an oligopoly, compared with a uniform price regime.
Proof. It is easy to show that

$$
\begin{equation*}
\pi^{d}(0, k)-\pi^{n d}(k, 0)=\frac{(1-b)(1+(k-2) b)\left(a_{1}-a_{2}\right)^{2}}{2(1+(k-1) b)(2+(k-3) b)^{2}}>0 . \tag{12}
\end{equation*}
$$

Proposition 2 indicates the same result derived by Adachi and Matsushima (2011) that a prisoner's dilemma will not occur. Besides, we have $\pi^{d}(0,1)-\pi^{n d}(1,0)=\left(a_{1}-a_{2}\right)^{2} / 8$ for $k=1$. This is the magnitude of the increase in profit of a monopoly after price discrimination when the demands of two separate markets are $q_{1}=a_{1}-p_{1}$ and $q_{2}=a_{2}-p_{2}$. Finally, we compare the welfare effects of the two regimes.

Proposition 3. Welfare must decrease under price discrimination in an oligopoly.
Proof. It is easy to show that

$$
\begin{equation*}
U^{n d}(k, 0)-U^{d}(0, k)=\left(U_{1}^{n d}+U_{2}^{n d}\right)-\left(U_{1}^{d}+U_{2}^{d}\right)=\frac{k(1-b)^{2}\left(a_{1}-a_{2}\right)^{2}}{4(1+(k-1) b)(2+(k-3) b)^{2}}>0 . \tag{13}
\end{equation*}
$$

Proposition 3 shows that welfare under third-degree price discrimination is low as compared with a uniform price regime. This is a known result in monopoly (see Varian, 1989) where with linear demand total output does not change with price discrimination. If the total output under uniform pricing is not lower than that under third degree price discrimination, then welfare is unambiguously larger under uniform pricing. The reason is that there is an allocation inefficiency associated with third-degree price discrimination. That is, different consumers pay different per-unit prices for a product, their marginal utilities will be unequal so that there exist unexploited opportunities for further trade. To overcome the welfare loss results from allocation inefficiency, Galera and Zaratiegui (2006) considers a duopoly with heterogenous firms, where price discrimination gives the low cost firm an opportunity to further exploit its competitive advantage. When this is the case, welfare can be improved even if output is decreased by practicing price discrimination.

## 3. Conclusions

We use a two-stage game to examine the domination of uniform price by third-degree price discrimination in an oligopoly. We suspect that if entry decisions made by firms are irrelevant, then third degree price discrimination is better than uniform pricing for firms in any finite games with complete information even if firms are asymmetric, the choice is not simultaneous, or not independent (coalition). First, each firm acts as a monopoly when all choice profiles of the other firms are given so that asymmetry is irrelevant. Second, when firms sequentially select prices or build

a coalition, third-degree price discrimination results in higher profits for all firms than under uniform pricing, as each of them solves the same profit maximization problem with fewer constraints.

In view of economic welfare, it depends on the tradeoff between the allocation inefficiency and production efficiency. To overcome the welfare loss associated with allocation inefficiency, we can consider asymmetric firms. Galera and Zaratiegui (2006) shows that if price discrimination gives the low production cost firm an opportunity to exploit its competitive advantage, then it is possible to enhance welfare. Schulz (1999) and Liu and Serfes (2010) argue that price discrimination can mitigate wasteful switching between firms, so that it in fact may improve efficiency in an asymmetric context. Adachi and Matsushima (2011) consider asymmetric product differentiation to show that price discrimination may be harmless from a social point of view.

When entry decisions are considered, the domination of uniform price by price discrimination may be changed. When firms find themselves in a prisoner's dilemma, and the increased profits of entering two markets are lower than the fixed costs incurred by entering the weak market, then it is beneficial for firms to give up the weak market and enter only one market. When this is the case, price discrimination is no more a dominant action compared with uniform pricing. As Azar (2003) indicates "Interestingly, while the constraint imposed is on pricing, firms prefer the game without price discrimination not because of its effect on prices but because of the equilibrium entry decisions."

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