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MATCHING MODELS AND HOUSING MARKETS: THE ROLE OF THE ZERO-PROFIT CONDITION

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Abstract
The recent and growing literature which has extended the use of search and matching models even to the housing market does not use the free entry or zero-profit assumption as a key condition for solving the equilibrium of the model. This is because a straightforward adaptation of the basic matching model to the housing market seems impossible. However, this short paper shows that the zero-profit condition can be easily reformulated to take the distinctive features of the housing market into account. Indeed, the zero-profit condition considers the possibility that a buyer can become a seller and vice versa, since it is used to find the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end.

Keywords: Housing Markets, Matching models, Zero-profit (or free-entry) condition

JEL classification: R21, R31, J63

1. Introduction

Recently, there has been much focus on formulising the behaviour of the housing market through the search and matching models usually used for the labour market (see, among others, Diaz and Jerez, 2009; Novy-Marx, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012). The housing market, like the labour market, clears not only through price but also through time and money that the parties spend on the market. Thus, the search and matching approach is certainly also suitable for this type of market.

However, all of these models do not use the free entry or zero-profit assumption as a key condition to solve the equilibrium of the model. Important differences between the labour and the housing markets seem to make a straightforward adaptation of the basic matching model (see Pissarides, 2000) to the study of the real estate market impossible. Precisely, the free entry or zero-profit condition for job creation in the labour market seems to have no counterpart in the housing market.

In fact, the assumption that vacancies in the housing market are created until the asset value of a vacant house is equal to zero may make sense if houses are supplied perfectly elastically by competitive house builders, in addition to being supplied by owners who no longer need them for occupation. However, the housing market is essentially a market for existing homes and the share of house sales accounted for by new construction is relatively small (Sheppard, 1999).

Nevertheless, the zero-profit condition can be easily reformulated to take the distinctive feature of the housing market into account, where buyers today are potential sellers tomorrow (Leung, Leong and Wong, 2006), and without introducing the construction sector interpretation. In particular, the zero-profit condition can be used to endogeneize the shares of buyers and sellers in the market. In short, since the value of a vacant house is closely linked to the value of being a seller, the zero-profit condition allows us to obtain the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end. When the value of a vacant house is equal to zero, in fact, no one will be willing to become a seller and thus the matching no longer occurs. In equilibrium, in fact, all the profit opportunities derived from buying/selling houses have been exploited.

The main implication of the paper for the related literature is that the use of the free-entry or zero-profit condition also in the housing market analysis allows to derive a direct relationship between market tightness and price similar to that obtained in a labour market matching model. In this way, the policy implications and the comparative statics become straightforward since an increase (decrease) in the selling price increases (decreases) the vacant houses on the market.[1] Also, as will be clear later, the developed model, where the zero-profit condition plays a key role, is able to provide a theoretical explanation for well-known empirical regularities in the housing markets, namely the trade-off between time-on-the-market and selling price and the empirical anomaly known as price dispersion. Thus, our main conclusion is that the free-entry or zero-profit condition represents a useful and correct guidance to analyse and formalise the price formation process in a housing market with search and matching frictions.

The rest of the paper is organised as follows: the next section presents a simple model of housing market where the zero-profit condition leads to equilibrium, while section 3 extends the model to study the transition process from seller (buyer) to buyer (seller).

2. A matching model of housing market

To make our point as simply as possible, we consider a basic matching model of housing market without rental and capital market. In the model, at any point in time, there is a mass of sellers (*s*) and a mass of buyers (*b*), and the population of buyers and sellers is normalised to the unit, i.e. $1 = s + b$.[2]

[1] Indeed, in the labour market the zero-profit condition gives a negative relationship between wage and market tightness; whereas, in the housing market it gives a positive relationship between house price and market frictions.

[2] There can certainly be households who are neither buyers nor sellers at a given point in time but this case does not change the main results of the analysis.

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The number of houses per capita h ranges between zero and a positive and exogenous number n, i.e. $0 < h < n$.[3] This number allows us to identify the type of economic agent in the market. Indeed, sellers are assumed to hold $h \geq 2$ houses of which $h-1$ are on the market. Hence, the vacant houses per capita are simple given by $h-1$ which is zero for the buyers. In the model, it is therefore possible for a buyer to become a seller and vice versa. In fact, when a seller (with two houses) manages to sell one house, s/he becomes a buyer, and then when s/he increases his/her inventory of houses to two by buying in the market, s/he becomes a seller again. Eventually, however, s/he stops trying to sell/buy further as s/he is either a seller or a buyer, but not both, at any point in time.

This matching model focuses on the transition process from seller (buyer) to buyer (seller), thus taking the homeownership market into account. In the homeownership market if a contract is legally binding, it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up. Therefore, the destruction rate of a specific buyer-seller match does not exist and the value of an occupied home for a seller is simply given by the selling price. As a result, the expected values of a vacant house V and of buying a house H are given by:[4]

$$rV = -c + q(\theta)\cdot[P - V] \tag{1}$$

$$rH = -e + g(\theta)\cdot[x - H - P] \tag{2}$$

where $\theta \equiv s/b$ is the housing market tightness which identifies the market frictions which prevent (or delay) the matching between the parties; standard hypothesis of constant returns to scale in the matching function, $m = m(s,b)$, is adopted and standard technical assumptions are postulated, namely $q(\theta) = m(s,b)/s$ and $g(\theta) = m(s,b)/b$ are, respectively, the instantaneous probability of filling a vacant house and of buying a home, with $\partial q(\theta)/\partial\theta < 0$, $\partial g(\theta)/\partial\theta > 0$, and $lim_{\theta\to 0} q(\theta) = lim_{\theta\to\infty} g(\theta) = \infty$, $lim_{\theta\to 0} g(\theta) = lim_{\theta\to\infty} q(\theta) = 0$. The terms c and e represent, respectively, the cost flows sustained by sellers and buyers during the search. When a match takes place, the risk neutral buyer gets a linear benefit x from the property and pays the sale price P to the seller.

First of all, note that the value of a vacant house is closely linked to the value of being a seller. Also, the number of sellers depends on market tightness:

$$\theta \equiv s/b = \frac{s}{1-s} \Rightarrow s = \frac{\theta}{\theta+1} \tag{3}$$

[3] Homelessness is irrelevant in the housing market analysis and it is not the equivalent of unemployed in the labour market. Buyers are generally not homeless, they are tenants or have other housing arrangements (e.g. young people living with their parents).
[4] Time is continuous; individuals are risk neutral, live infinitely and discount the future at the exogenous and positive interest rate r.

Hence, the zero-profit condition can be reformulated in the housing market matching model to find the share of sellers and buyers in equilibrium. Eventually, in fact, the person stops trying to sell/buy further as s/he is either a seller or a buyer, but not both, at any point in time. Indeed, the transition process from seller (buyer) to buyer (seller) comes to an end when the value of a vacant house is equal to zero. In this case, in fact, no one will be willing to become a seller and thus the matching no longer occurs. Formally,

$$V = 0 \Rightarrow \frac{c}{q(\theta)} = P \Rightarrow \theta^* \Rightarrow s^*, b^* \tag{4}$$

In equilibrium, in fact, all the profit opportunities derived from buying/selling houses have been exploited. It follows that, given the equilibrium value of market tightness, we find the (total) share of sellers and buyers in the housing market.

Also, the zero-profit condition (4) gives a positive relationship between selling price and market tightness: an increase in P increases θ, since the (expected) time-on-the-market, i.e. the inverse of the probability of filling a vacancy $[q(\theta)]^{-1}$, is increasing in θ. The sale price P is instead obtained by the *Nash bargaining solution* usually used for decentralised markets:[5]

$$P = argmax\left\{(P - V)^{\gamma} \cdot (x - H - P)^{1-\gamma}\right\} \Rightarrow P = \gamma \cdot (x - H) \tag{5}$$

where $0 < \gamma < 1$ is the share of the bargaining power of sellers. Note that the selling price depends negatively on market tightness (the well-known *congestion externalities* effect on the supply side), since H is (obviously) increasing in θ. Thus, only one long-term equilibrium with a positive value of P and θ exists in the model. This testable proposition is made possible by a model with a positive relationship between housing prices and time on the market (which starts from the origin of the axes) and a downward sloping price function (with positive vertical intercept). Therefore, the model is able to reproduce the observed joint behaviour of prices and time-on-the-market: in fact, the house with a higher price has a longer time on the market; whereas, the longer the time-on-the-market the lower the sale price (Leung, Leong, and Chan, 2002).

Furthermore, by assuming different bargaining strengths and search costs, this model can explain price dispersion in housing prices (Leung, Leong and Wong, 2006): in fact, in that case, housing prices would be different even for similar houses (i.e. houses which give the same benefit x). Although these empirical facts have also been explained by models which use a different condition, like the buyers' free entry (Leung and Zhang, 2011), the zero-profit condition allows to obtain a solution which characterise the direct relationship between market tightness and house price. In this way, the known features of the housing market can be explained in a straightforward way (as this note clearly shows).

[5] The derivation of the solution to the bargaining problem is too standard to be repeated here.

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3. The transition process

Although homelessness seems equivalent to unemployment, changes in the housing markets are very different from shifts in the labour market (Wheaton, 1990). Hence, in the housing market is more interesting to study the transition from seller to buyer (and vice versa) rather than the dynamic in and out of the homelessness.

The model developed in the previous section predicts the existence of different types of sellers but it does not study the transition process in the housing market. Hence, this section extends the model to study the transition process from seller to buyer (and vice versa) and take into account the different kinds of sellers. It follows that $s = \sum_{h=2}^{n} s(h)$.

As regards the sellers who hold more than two houses, namely $s(h > 2)$, the dynamics are very simple since in steady state we get that $s(h+1) = s(h)$, $\forall h > 2$. In words, when a seller with *h+1* houses manages to sell one house, s/he becomes a seller with *h* houses, and when a seller with *h* houses manages to sell one house, s/he becomes a seller with *h – 1* houses. By assuming an inflow equal to κ for $s(h = n)$, we can write the total share of sellers as follows:[6]

$$s = s(2) + \sum_{h=3}^{n} s(h) \Rightarrow s = s(2) + (n-2)\cdot\frac{\kappa}{q(\theta)} \qquad (6)$$

Obviously, more interesting is the dynamics of threshold value, namely the dynamics of sellers who hold two houses (the transition process from seller to buyer and vice versa):
$\dot{s}(2) = b\cdot g(\theta) - [s(2) - s(3)]\cdot q(\theta)$
where $b\cdot g(\theta)$ are the buyers who become sellers, $s(2)\cdot q(\theta)$ are the sellers who become buyers and $s(3)\cdot q(\theta)$ are the sellers with *3* houses who become sellers with *2* houses. In steady state we obtain:

$$\dot{s}(2) = 0 \Rightarrow s(2) = b\cdot\frac{g(\theta)}{q(\theta)} + \frac{\kappa}{q(\theta)} \qquad (7)$$

Therefore, given the (total) share of sellers (*s*) and buyers (*b*), we get a system of two equations (6 – 7), in two unknowns, $s(2)$ and κ. Hence, it is straightforward to show that sufficient condition for

[6] Suppose, for example, that $n = 4$. In this case we get: $\dot{s}(4) = \kappa - s(4)\cdot q(\theta) \Rightarrow \dot{s}(4) = 0 \Rightarrow \kappa / q(\theta) = s(4)$, and $\dot{s}(3) = [s(4) - s(3)]\cdot q(\theta) \Rightarrow \dot{s}(3) = 0 \Rightarrow s(4) = s(3) = \kappa / q(\theta)$.

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the existence of an interior equilibrium is $s - b \cdot \frac{g(\theta)}{q(\theta)} > 0$, i.e. $q(\theta)$ is sufficiently higher than $g(\theta)$. In words, the perspective of becoming seller is very attractive. Indeed, in the model the purchase is directed to the sale of the property.

4. Conclusions

Almost all the matching models of the housing market do not use the free entry or zero-profit assumption as a key condition to solve the equilibrium of the model. This is because a straightforward adaptation of the basic matching model to the housing market seems impossible. In this short paper we show that the zero-profit condition can be easily reformulated to take the distinctive features of the housing market into account. Although the model used is rather general and simple, the zero-profit condition has a clear economic meaning since it is used to find the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end.

Thus, the free-entry or zero-profit condition represents a useful and correct rule which allows to derive a direct relationship between house price and market tightness. Policy implications and the comparative statics thus become straightforward since a change in the selling price (market tightness) immediately affects market tightness (selling price). For example, by adding a tax on property sale one can study the effects of taxation on both selling price and vacant houses on the market. Finally, the zero-profit condition helps to reproduce stylised facts of housing markets, such as the trade-off between time-on-the-market and selling price and the empirical anomaly known as price dispersion, in a straightforward way.

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