The Economic Research Guardian – Vol. 4(2)2014
Semi-annual Online Journal, www.ecrg.ro
ISSN: 2247-8531, ISSN-L: 2247-8531
Econ Res Guard 4(2): 121-141

TRADABLE, NON-TRADABLE AND EDUCATION SECTORS IN A MULTI-COUNTRY ECONOMIC GROWTH MODEL WITH ENDOGENOUS WEALTH AND HUMAN CAPITAL

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Abstract

This study deals with international economic interactions with capital accumulation, human capital accumulation, economic structure and international trade by integrating the Solow growth model, the Uzawa two-sector growth model, the Uzawa-Lucas two-sector growth model, and the Oniki–Uzawa trade model within a comprehensive framework. In addition to learning by education in the Uzawa-Lucas model, we also consider Arrow's learning by producing, and Zhang's learning by consuming (creative learning) in the human capital accumulation equation. The model is built for any number of national economies and each national economy consists of one tradable, one non-tradable and one education sector. National economies are different in propensities to save, to obtain education and to consume, and in learning abilities. We show that the dynamics of the J-country world economy can be described by 2J differential equations. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also demonstrate how changes in the propensity to obtain education, the population, the propensity to save, and the education sector's total productivity affect global economic development.

Keywords: Trade pattern, Education, Non-tradable, Wealth accumulation

JEL classification: E13, F11, I25

1. Introduction

This study is primarily concerned with dynamic interdependence among wealth and physical capital accumulation, human capital accumulation, and trade patterns in a multi-country neoclassical growth theory framework. The growth mechanism of physical accumulation is based on the Solow growth model. We describe international trade on the basis of the dynamic model with accumulating capital developed by Oniki and Uzawa and others (for instance, Oniki and Uzawa, 1965; Frenkel and Razin, 1987; Sorger, 2002; and Nishimura and Shimomura, 2002). Although there are also some other trade models with endogenous capital accumulation, most of trade models with endogenous capital are still either limited to two-country or small open economies without taking account of endogenous human capital (for instance, Grossman and Helpman, 1991). Moreover, we use tradable good and non-tradable

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rather than capital goods and consumer goods as in the Oniki-Uzawa model. A high share of GDP in modern economies non-tradable. In contemporary literature of open economies nontradable goods have received a great attention. Distinction between tradable good and non-tradable good is significant for explaining the terms of trade (Mendoza, 1995), for explaining the exchange rate (Stulz, 1987; Stockman and Dellas, 1989; Backus and Smith, 1993; Rogoff, 2002); for dealing with current account dynamics (Edwards, 1989), or for solving the home premium puzzle (Baxter et al., 1998; Pesenti and van Wincoop, 2002). A reason for this distinction is given by Backus and Smith (1993:1) as follows: "The mechanism is fairly simple. "Although the law of one price holds, in the sense that each good sells for a single price in all countries, PPP may not: price indexes combine prices of both traded and nontraded goods, and because the latter are sold in only one country their prices, and hence price indexes, may differ across countries." Stockman and Tesar (1995) observe that the tradable sector is generally more volatile.

The study is also concerned with differences in human capital between countries. As observed by Easterlin (1981), in 1850 there were few people outside North-Western Europe and North America who had any formal education. In modern economies, human capital is considered a key determinant of economic growth (Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012; and Barro and Lee, 2013; Hanushek *et al.* 2014). This study considers three sources of learning – education in the Uzawa-Lucas model, Arrow's learning by doing, and Zhang's learning by consuming (which include leisure, family conditions, travels and readings at leisure, and so on). There is an extensive literature on education and economic growth. Mincer (1974) published the seminal work in 1974 on estimating the impact of education on earnings. As point out by Tilak (1989), spread education can substantially reduce inequality within countries. Could *et al.* (2001) study dynamics of wage inequality within and between industries and education groups in the past few decades. It is shown that inequality growth within educated workers is determined more by changes in the composition and return to ability (which is closely related to education). Tselios (2008) identifies a positive relationship between income and educational inequalities in the regions of the European Union. Fleisher *et al.* (2011) find that an additional year of schooling raises marginal product by 30.1 percent, and the CEO's education increases TFP for foreign-invested firms. Zhu (2011) find that the heterogeneity in schooling returns falls from 1995 to 2002 for both genders in urban China, although their rates of education return have increased substantially. There is also a large number of the theoretical literature on endogenous knowledge and economic growth (Romer, 1986; Lucas, 1988; Grossman and Helpman, 1991; and Aghion and Howitt, 1998). The first formal dynamic growth model with education was proposed by Uzawa (1965). The Uzawa-Lucas model has a strong impact on the recent development of the literature (e.g., Galor and Zeira, 1993; Maoz and Moav, 1999; Galor and Moav, 2004; Fender and Wang, 2003; Erosa *et al.* 2010). A main deviation of our approach from the previous models is that we derive demand of education in an alternative approach to the typical Ramsey approach. Another problem in the Uzawa-Lucas model and many of their extensions and generalizations is that all skills and human capital are formed due to formal schooling. But it is commonly held that much of human capital may be accumulated in family and many other social and economic activities. Ignoring non-school determinants in human capital accumulation factors may mislead us in properly understanding the role of formal education in economic development (e.g., Erosa et al. 2010; Schoellman, 2012; Kaarsen, 2014). In order to more properly modelling human capital accumulation, this study takes account of Arrow's learning by doing (Arrow, 1962) and Zhang's creative leisure (Zhang, 2007) in modeling human capital accumulation.

This paper develops a multi-country growth trade model with economic structure, treating the global economy as an integrated whole. The economic system is built on the basis of the Solow model, the Uzawa two-sector model, the Oniki-Uzawa trade model, and the Uzawa-Lucas two-sector growth model. Different from the growth models with the Ramsey approach, we use an alternative utility function proposed by Zhang (1993) to determine saving and consumption. We analyze trade issues within the framework of a simple international macroeconomic growth model with perfect capital mobility. The model in this study is a further development of the two models by Zhang. Zhang (2012) proposed a multi-country model with capital accumulation and knowledge without education. Although Zhang (2013) introduced education into a neoclassical growth model, the model was limited to a national economy. This paper develops an international growth model with education. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the motion of the global economy. Section 4 carries out comparative dynamic analysis to examine the impact of changes in some parameters on the motion of the global economy. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. The model

The model in this study is developed within the framework of the neoclassical growth theory with international trade. The world economy consists of multiple countries, indexed by $j = 1, \ldots, J$. Country j has a fixed population, $\bar{N}_j$, ($j = 1, \ldots, J$). Each country has three sectors: one tradable good sector, one non-tradable goods sector and one education sector. We assume that all the economy can produce a homogenous tradable commodity (see also Ikeda and Ono, 1992). The commodity is like the commodity in the Solow model which can be consumed and invested. Each economy can thus produce one (durable) good in the global economy and one non-tradable (domestic) good. Households own assets of the economy and distribute their incomes to consume, to receive education, and to save. Exchanges take place in perfectly competitive markets. Production factor are fully utilized at every moment. Saving is undertaken only by households. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. Let price be measured in terms of the tradable good and the price of the good be unit. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the jth country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$. Capital depreciates at a constant exponential rate δ_{jk}, being independent of the manner of use within each country. Depreciation rates may vary between countries. Let $p_j(t)$ and $p_{js}(t)$ denote the price of education and the price of non-tradable good. We use subscript index, i, s and e to stand for tradable good sector, non-tradable good sector, and education sector, respectively, in country j. We use $N_{jm}(t)$ and $K_{jm}(t)$ to stand for the labor force and capital stocks employed by sector m in country j. Let $F_{jm}(t)$ stand for the output level of sector m in country j.

The labor supply

The aggregated labor force $N_j(t)$ of country j is given by

$$N_j(t) = H_j^{m_j}(t) T_j(t) \bar{N}_j, \tag{1}$$

where $H_j(t)$ and $T_j(t)$ are respectively the level of human capital and work time in country j. Here, m_j is a positive parameter measuring how household j effectively applies human capital.

Marking clearing conditions

We use $K(t)$ to stand for the capital stocks of the world economy. The total capital stock employed by country j, $K_j(t)$, is allocated between the three sectors. We use $\bar{K}_j(t)$ to stand for the wealth owned by country j. Due to the assumed full employment of labor and capital, we have

$$K_{ji}(t) + K_{js}(t) + K_{je}(t) = K_j(t), \quad N_{ji}(t) + N_{js}(t) + N_{je}(t) = N_j(t).$$

We rewrite the above relations as follows

$$n_{ji}(t)k_{ji}(t) + n_{js}(t)k_{js}(t) + n_{je}(t)k_{je}(t) = k_j(t), \quad n_{ji}(t) + n_{js}(t) + n_{je}(t) = 1, \tag{2}$$

in which

$$k_{jq}(t) \equiv \frac{K_{jq}(t)}{N_{jq}(t)}, \quad n_{jq}(t) \equiv \frac{N_{jq}(t)}{N_j(t)}, \quad k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad q = i, s, e.$$

Production functions

We use the conventional production function to describe a relationship between inputs and output. The functions, $F_{jq}(t)$, are specified by

$$F_{jq}(t) = A_{jq} K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \quad A_{jq}, \alpha_{jq}, \beta_{jq} > 0, \quad \alpha_{jq} + \beta_{jq} = 1, \tag{3}$$

where A_{jq}, α_{jq}, and β_{jq} are positive parameters.

Marginal conditions

Each production sector chooses the two variables $K_{ji}(t)$ and $N_{ji}(t)$ to maximize its profit. The marginal conditions are

$$r(t) + \delta_{jk} = \alpha_{ji} A_{ji} k_{ji}^{-\beta_{ji}}(t), \quad w_j(t) = \beta_{ji} A_{ji} k_{ji}^{\alpha_{ji}}(t), \tag{4}$$

$$r(t) + \delta_{jk} = \alpha_{js} A_{js} p_{js}(t) k_{js}^{-\beta_{js}}(t), \quad w_j(t) = \beta_{js} A_{js} p_{js}(t) k_{js}^{\alpha_{js}}(t), \tag{5}$$

where δ_{jk} is depreciation rate of physical capital.

Education sector

Students are supposed to pay the education fee $p_j(t)$ per unit of time in country. The education sector pays teachers and capital with the market rates. The cost of the education sector is given by $w_j(t)N_{je}(t) + (r(t) + \delta_{jk})K_{je}(t)$. The total education service is measured by the total education time received by the population. The production function of the education sector is assumed to be a function of $K_{je}(t)$ and $N_{je}(t)$. We specify the production function of the education sector as follows

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$$F_{je}(t) = A_{je} K_{je}^{\alpha_{je}}(t) N_{je}^{\beta_{je}}(t), \ \alpha_{ej}, \beta_{je} > 0, \ \alpha_{je} + \beta_{je} = 1, \tag{6}$$

where A_{je}, α_{je} and β_{je} are positive parameters. For given $p_j(t)$, $H_{jq}(t)$, $r(t)$, and $w_j(t)$, the marginal conditions for the education sector are given by

$$r(t) + \delta_{jk} = \alpha_{je} A_{je} p_j(t) k_e^{-\beta_e}(t), \ w_j(t) = \beta_{je} A_{je} p_j(t) k_{je}^{\alpha_{je}}(t). \tag{7}$$

Consumer behaviors and wealth dynamics

This study uses Zhang's utility function to describe behavior of households (Zhang, 1993). Let $\bar{k}_j(t)$ stand for wealth of household j. Per household's current income from the interest payment $r(t)\bar{k}_j(t)$ and the wage payment $H_j^{m_j}(t) T_j(t) w_j(t)$ is given by

$$y_j(t) = r(t)\bar{k}_j(t) + H_j^{m_j}(t) T_j(t) w_j(t).$$

We call $y_j(t)$ the current income. The per capita disposable income is then given by

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t) = (1 + r(t))\bar{k}_j(t) + H_j^{m_j}(t) T_j(t) w_j(t). \tag{8}$$

Let T_0 stand for the time spent on education. We assume that the total available time is distributed between education and work. The consumer is faced with the following time constraint

$$T_j(t) + T_{je}(t) = T_0, \tag{9}$$

where T_0 is the total available time. The consumer is faced with the following budget constraint

$$c_j(t) + p_{js}(t) c_{js}(t) + s_j(t) + p_j(t) T_{je}(t) = \hat{y}_j(t). \tag{10}$$

Substituting (9) into (8) yields

$$c_j(t) + p_{js}(t) c_{js}(t) + s_j(t) + \bar{p}_j(t) T_{je}(t) = \bar{y}_j(t), \tag{11}$$

where

$$\bar{p}_j(t) \equiv p_j(t) + \bar{w}_j(t), \ \bar{w}_j(t) \equiv H_j^{m_j}(t) w_j(t), \ \bar{y}_j(t) \equiv (1 + r(t))\bar{k}_j(t) + T_0 \bar{w}_j(t). \tag{12}$$

The consumer's utility is a function of the consumption level of tradable good $c_j(t)$, the consumption level of non-tradable good $c_{js}(t)$, the education time $T_{je}(t)$, and the level of saving $s_j(t)$, as follows

$$U_j(t) = c_j^{\xi_{j0}}(t) c_{js}^{\gamma_{j0}}(t) s_j^{\lambda_{j0}}(t) T_{je}^{\eta_{j0}}(t), \ \xi_{j0}, \gamma_{j0}, \lambda_{j0}, \eta_{j0} > 0,$$

where ξ_{j0} is called the propensity to consume tradable good, γ_{j0} the propensity to consume non-tradable good, λ_{j0} the propensity to own wealth, and η_{j0} the propensity to receive education. Maximizing $U_j(t)$ subject to the budget constraint yields

$$c_j(t) = \xi_j \bar{y}_j(t), \ p_{js}(t) c_{js}(t) = \gamma_j \bar{y}_j(t), \ s_j(t) = \lambda_j \bar{y}_j(t), \ \bar{p}_j(t) T_{je}(t) = \eta_j \bar{y}_j(t), \tag{13}$$

where

$$\xi_j \equiv \rho_j \xi_{j0}, \ \gamma_j \equiv \rho_j \gamma_{j0}, \ \lambda_j \equiv \rho_j \lambda_{j0}, \ \eta_j \equiv \rho_j \eta_{j0}, \ \rho_j \equiv \frac{1}{\xi_{j0} + \gamma_{j0} + \lambda_{j0} + \eta_{j0}}.$$

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Wealth accumulation
According to the definitions of $s_j(t)$, we have

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{14}$$

Human capital accumulation
According to the conclusions by Hanushek and Woessmann (2008), the cognitive skills of the population, rather than mere school attainment, are strongly related to economic growth, individual earnings, and the distribution. A recent study by Kaarsen (2014) on estimating differences in education quality finds: "there are large differences in education quality across countries. One year of schooling in the U.S. corresponds to 3 or even 4 years of schooling in many developing countries." Hanushek (2013: 204) point out: "Developing countries have made considerable progress in closing the gap with developed countries in terms of school attainment, but recent research has underscored the importance of cognitive skills for economic growth. … Without improving school quality, developing countries will find it difficult to improve their long run economic performance." As human capital is not only affecting by schooling, it is necessary to introduce other possible determinants. We take account of three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow (1962) first introduced learning by doing into growth theory; Uzawa (1965) took account of trade-offs between investment in education and capital accumulation, and Zhang (2007) introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory. Following Zhang (2007), we propose the following human capital accumulation equation

$$\dot{H}_j = \frac{\upsilon_{je} F_{je}^{a_{je}} \left(H_j^{m_j} T_{je} \bar{N}_j\right)^{b_{je}}}{H_j^{\pi_{je}} \bar{N}_j} + \frac{\upsilon_{ji} F_{ji}^{a_{ji}}}{H_j^{\pi_{ji}} \bar{N}_j} + \frac{\upsilon_{jh} c_j^{a_{jh}}}{H_j^{\pi_{jh}}} + \frac{\upsilon_{js} c_{js}^{a_{js}}}{H_j^{\pi_{js}}} - \delta_{jh} H_j, \tag{15}$$

where δ_{jh} (> 0) is the depreciation rate of human capital, $\upsilon_{je}, \upsilon_{ji}, \upsilon_{jh}, a_{je}$, b_{je}, a_{ji}, and a_{jh} are non-negative parameters. The signs of the parameters π_{je}, π_{ji}, and π_{jh} are not specified as they may be either negative or positive.

Market clearing in education markets
For the education sector, the demand and supply balances in each country

$$T_{je}(t)\bar{N}_j = F_{je}(t). \tag{16}$$

Market clearing in non-tradable good markets
For each country, the demand for non-tradable good equals the supply at any point time

$$c_{js}(t)\bar{N}_j = F_{js}(t). \tag{17}$$

Market clearing in tradable good markets
The total capital stocks in international markets employed by the production sectors is equal to the total wealth owned by all the countries. That is

$$K(t) \equiv \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j. \tag{18}$$

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The world production is equal to the world consumption and world net savings. That is

$$C(t)+S(t)-K(t)+\sum_{j=1}^{J}\delta_{kj}K_j(t)=F(t), \tag{19}$$

where

$$C(t)\equiv\sum_{j=1}^{J}c_j(t)\bar{N}_j,\ \ S(t)\equiv\sum_{j=1}^{J}s_j(t)\bar{N}_j,\ \ F(t)\equiv\sum_{j=1}^{J}F_{ji}(t).$$

International trade

The trade balances of the economies are given by

$$E_j(t)=\left(\bar{K}_j(t)-K_j(t)\right)r(t). \tag{20}$$

We built the model with trade, economic growth, physical and human capital accumulation in the world economy in which the domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile and labor is internationally immobile. The model synthesizes main ideas in economic growth theory and trade theory in a comprehensive framework.

3. The dynamics and equilibrium

Before examining the dynamic properties of the system, we show that the dynamics of J national economies can be expressed by $2J$ differential equations.

Lemma

The motion of $2J$ variables, $\{\bar{k}_j(t)\}$, $k_{1i}(t)$, and $(H_j(t))$, where $\{\bar{k}_j(t)\}=(\bar{k}_2(t),\ ...,\ \bar{k}_J(t))$, is given by the following differential equations

$$\dot{k}_{1i}(t)=\bar{\Lambda}_1\left(k_{1i}(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right),$$
$$\dot{\bar{k}}_j(t)=\bar{\Lambda}_j\left(k_{1i}(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right),\ \ j=2,\ ...,\ J,$$
$$\dot{H}_j(t)=\Lambda_j\left(k_{1i}(t),\ (H_j(t)),\ \{\bar{k}_j(t)\}\right), \tag{21}$$

where $\bar{\Lambda}_j(t)$ and $\Lambda_j(t)$ are functions of $\{\bar{k}_j(t)\}$, $k_{1i}(t)$, and $(H_j(t))$, defined in the appendix.

The values of the other variables are given as functions of $\{\bar{k}_j(t)\}$, $k_{1i}(t)$, and $(H_j(t))$, at any point in time by the following procedure: $k_{ji}(t)$ by (A3) $\rightarrow$ $k_{js}(t)$ by (A1) $\rightarrow$ $k_{je}(t)$ by (A4) $\rightarrow$ $r(t)$ and $w_j(t)$ by (4) $\rightarrow$ $p_j(t)$ by (A5) $\rightarrow$ $\bar{k}_1(t)$ by (A14) $\rightarrow$ $K_j(t)$ by (A13) $\rightarrow$ $\bar{w}_j(t)$, $\bar{p}_j(t)$ and $\bar{y}_j(t)$ by (12) $\rightarrow$ $k_j(t)$ by (A12) $\rightarrow$ $T_j(t)$ by (A11) $\rightarrow$ $T_{je}(t)$ by (13) $\rightarrow$ $N_j(t)=T_j(t)H_j^{m_j}(t)\bar{N}_j$ $\rightarrow$ $n_{ji}(t)$ and $n_{js}(t)$ by (A7) $\rightarrow$ $n_{je}(t)$ by (A6) $\rightarrow$ $N_{jq}(t)=n_{jq}(t)N_j(t)$, $q=i,\ s,\ e$ $\rightarrow$ $K_{jq}(t)=k_{jq}(t)N_{jq}(t)$ $\rightarrow$ $F_{ji}(t)$ and $F_{js}(t)$ by (3) $\rightarrow$ $F_{je}(t)$ by (6) $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (13) $\rightarrow$ $E_j(t)=\left(\bar{K}_j(t)-K_j(t)\right)\bar{N}_j$.

For simulation, we specify values of the parameters. We consider the world consists of three national economies, i.e., $J = 3$. We specify

$$N_1 = 5,\ N_2 = 10,\ N_3 = 20,\ T_0 = 1,\ m_1 = 0.9,\ m_2 = 0.85,\ m_3 = 0.75. \tag{22}$$

Country $1, 2$ and $3's$ populations are respectively $5, 10$ and 20. Country 3 has the largest population. Country 1 uses human capital most effectively and Country 2 next. The parameters in the production functions and physical capital depreciation rates of the three economies are

$$\begin{gathered} A_{1i} = 1.3,\ A_{1s} = 1.2,\ A_{1e} = 1.1,\ \alpha_{1i} = 0.32,\ \alpha_{1s} = 0.35,\ \alpha_{1e} = 0.45,\ A_{2i} = 1.25, \\ A_{2s} = 1.1,\ A_{2e} = 1,\ \alpha_{2i} = 0.32,\ \alpha_{2s} = 0.36,\ \alpha_{2e} = 0.45,\ A_{3i} = 1.2,\ A_{3s} = 1, \\ A_{3e} = 0.9,\ \alpha_{3i} = 0.32,\ \alpha_{3s} = 0.37,\ \alpha_{3e} = 0.45,\ \delta_{1k} = 0.06,\ \delta_{2k} = 0.05,\ \delta_{3k} = 0.05. \end{gathered} \tag{23}$$

The total factor productivities are different between three economies. Country 1's total factor productivity is highest and Country 3's total factor productivity is lowest. We call countries 1, 2 and 3 respectively as highly developed, developed, and lowly developed economies (HDE, DE, LDE). The output elasticities with respect labor and capital also vary between countries. We specify the values of the parameters, α_j, in the Cobb-Douglas productions approximately equal to 0.3. The depreciation rate of physical capital is specified near 0.05. We specify the household preferences of the three economies as

$$\begin{gathered} \gamma_{10} = 0.06,\ \eta_{10} = 0.07,\ \xi_{10} = 0.1,\ \lambda_{10} = 0.73,\ \gamma_{20} = 0.06,\ \eta_{20} = 0.06,\ \xi_{20} = 0.1, \\ \lambda_{20} = 0.07,\ \gamma_{30} = 0.06,\ \eta_{30} = 0.05,\ \xi_{30} = 0.1,\ \lambda_{30} = 0.68. \end{gathered} \tag{24}$$

The HDE's propensity to save is 0.73, the DE's propensity to save is 0.7, and the LDE's propensity to save is 0.6. We specify the human capital accumulation as follows

$$\begin{gathered} v_{1e} = 1.2,\ v_{1i} = 3,\ v_{1h} = 1.2,\ v_{1s} = 1.2,\ a_{1e} = 0.3,\ b_{1e} = 0.5,\ a_{1i} = 0.4,\ a_{1h} = 0.1,\ a_{1s} = 0.3, \\ \pi_{1e} = 0.1,\ \pi_{1i} = 0.7,\ \pi_{1h} = 0.1,\ \pi_{1s} = 0.1,\ \delta_{1h} = 0.05,\ v_{2e} = 1.1,\ v_{2i} = 2.7,\ v_{2h} = 1,\ v_{2s} = 1, \\ a_{2e} = 0.3,\ b_{2e} = 0.5,\ a_{2i} = 0.4,\ a_{2h} = 0.1,\ a_{2s} = 0.3,\ \pi_{2e} = 0.1,\ \pi_{2i} = 0.7,\ \pi_{2h} = 0.1,\ \pi_{2s} = 0.1, \\ \delta_{2h} = 0.05,\ v_{3e} = 1,\ v_{3i} = 2.5,\ v_{3h} = 1,\ v_{3s} = 1,\ a_{3e} = 0.3,\ b_{3e} = 0.5,\ a_{3i} = 0.4, \\ a_{3h} = 0.1,\ a_{3s} = 0.3,\ \pi_{3e} = 0.1,\ \pi_{3i} = 0.7,\ \pi_{3h} = 0.1,\ \pi_{3s} = 0.1,\ \delta_{3h} = 0.05. \end{gathered} \tag{25}$$

The human capital depreciation rates of the three economies are equal. The HDE's human capital accumulation efficiency due to education v_{1e} is highest, the DE's is next, and the LDE's is lowest. Similarly the specified values in (24) imply that the HDE is most effective in accumulating human capital, the DE is next, and the LDE is least effective. As we already provided the procedure to follow the motion of each variable in the system, it is straightforward to plot the motion with computer. We specify the initial conditions as follows:

$$k_{1i}(0) = 7.4,\ \bar{k}_2(0) = 230,\ \bar{k}_3(0) = 139,\ H_1(0) = 92,\ H_2(0) = 84,\ H_3(0) = 67.$$

The motion of the system is given in Figure 1. In the figure the GDPs per capita and the global GDP are defined as follows

$$g_j \equiv \frac{F_{ji} + p_{js} F_{js} + p_{je} F_{je}}{\bar{N}_j},\ Y \equiv \sum_j \left(F_{ji} + p_{js} F_{js} + p_{je} F_{je}\right).$$

Because of the chosen initial values, the HDE's GDP per capita and human capital are enhanced over time, the other two countries' GDP per capita and human capital are lowered. The HDE's total labor force and capital stocks employed are augmented and the other two economies' total labor forces and capital stocks employed are lowered. This implies that the GDP per capita gaps between the HDE and

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Semi-annual Online Journal, www.ecrg.ro
ISSN: 2247-8531, ISSN-L: 2247-8531
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the other two economies are enlarged over time. The prices of education and non-tradable goods fall slightly in the three economies. The rate of interest falls and the wage rates are increased. The HDE's wage income per capita is increased, and the other two economies' wage incomes per capita are reduced. The labor and capital are redistributed between the three sectors in each economy over time. The system approaches an equilibrium point in the long term. Our simulation shows that the representative household from a country with higher GDP per capita spends more time on education.

It should be noted that much of the discussion of income convergence in the literature of economic growth and development is based on the insights from analyzing models of closed economies (Barro and Sala-i-Martin, 1995). It is obviously strange to discuss issues related to global income and wealth convergence with a framework without international interactions. The reason for this is that there are few growth models with endogenous wealth and trade on the basis of microeconomic foundation. As shown in Figure 1, different countries will not experience convergence in per capita income, consumption and wealth in the long term as they are different in preferences and total productivities. A recent empirical study on the determinants of economic growth and investment with a panel of around 100 countries from 1960 to 1995, Barro (2013: 327) observes that "The data reveal a pattern of conditional convergence in the sense that the growth rate of per capita GDP is inversely related to the starting level of per capita GDP, holding fixed measures of government policies and institutions, initial stocks of human capital, and the character of the national population. With respect to education, growth is positively related to the starting level of average years of school attainment of adult males at the secondary and higher levels." Our model shows different patterns. From Figure 1 we observe that the system becomes stationary in the long term. The six eigenvalues are

$$\{-0.23, -0.22, -0.20, -0.04, -0.04, -0.04\}.$$

This implies that the world economy is stable. Following Lemma 1 under (15), we calculate the equilibrium values of the variables as follows:

$$\begin{pmatrix} r \\ K \\ Y \end{pmatrix} = \begin{pmatrix} 0.041 \\ 7151 \\ 2017 \end{pmatrix}, \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 0.87 \\ 0.91 \\ 0.98 \end{pmatrix}, \begin{pmatrix} p_{1s} \\ p_{2s} \\ p_{3s} \end{pmatrix} = \begin{pmatrix} 1.02 \\ 1.04 \\ 1.07 \end{pmatrix}, \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} 114.6 \\ 69.8 \\ 37.3 \end{pmatrix}, \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} 11.86 \\ -2.92 \\ -8.95 \end{pmatrix},$$

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 223.6 \\ 273.6 \\ 308.8 \end{pmatrix}, \begin{pmatrix} n_{1e} \\ n_{2e} \\ n_{3e} \end{pmatrix} = \begin{pmatrix} 0.0022 \\ 0.0033 \\ 0.0058 \end{pmatrix}, \begin{pmatrix} n_{1s} \\ n_{2s} \\ n_{3e} \end{pmatrix} = \begin{pmatrix} 0.30 \\ 0.29 \\ 0.28 \end{pmatrix}, \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 1864 \\ 2544 \\ 2743 \end{pmatrix}, \begin{pmatrix} \bar{K}_1 \\ \bar{K}_2 \\ \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 2151 \\ 2474 \\ 2526 \end{pmatrix},$$

$$\begin{pmatrix} k_{1i} \\ k_{2i} \\ k_{3i} \end{pmatrix} = \begin{pmatrix} 7.98 \\ 8.78 \\ 8.27 \end{pmatrix}, \begin{pmatrix} k_{1s} \\ k_{2s} \\ k_{3s} \end{pmatrix} = \begin{pmatrix} 9.13 \\ 10.5 \\ 10.3 \end{pmatrix}, \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 395 \\ 484 \\ 518 \end{pmatrix}, \begin{pmatrix} F_{1e} \\ F_{2e} \\ F_{3e} \end{pmatrix} = \begin{pmatrix} 1.74 \\ 3.10 \\ 5.34 \end{pmatrix}, \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 174.1 \\ 204.3 \\ 207.6 \end{pmatrix},$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} 44.7 \\ 27.4 \\ 15.4 \end{pmatrix}, \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} 109.7 \\ 75.9 \\ 58.2 \end{pmatrix}, \begin{pmatrix} T_{1e} \\ T_{2e} \\ T_{3e} \end{pmatrix} = \begin{pmatrix} 0.35 \\ 0.31 \\ 0.27 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 58.9 \\ 35.3 \\ 18.6 \end{pmatrix}, \begin{pmatrix} c_{1s} \\ c_{2s} \\ c_{3s} \end{pmatrix} = \begin{pmatrix} 34.8 \\ 20.4 \\ 10.4 \end{pmatrix}, \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{pmatrix} = \begin{pmatrix} 430.3 \\ 247.4 \\ 126.3 \end{pmatrix}.$$

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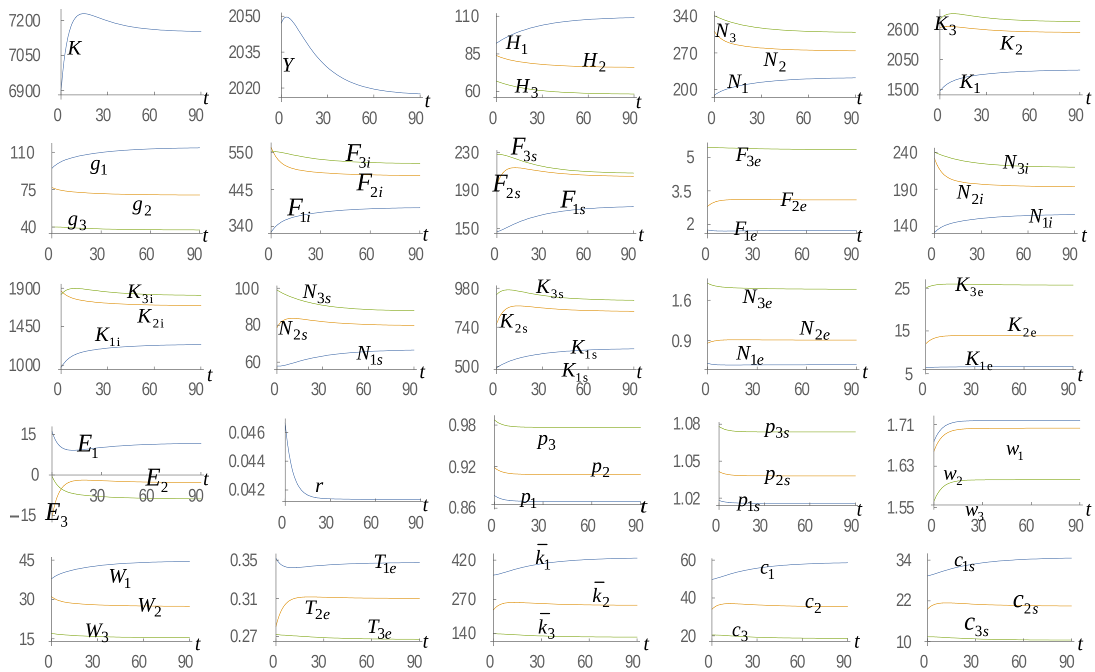

Figure 1 - The Motion of the Global Economy

4. Comparative dynamic analysis

We simulated the motion of the dynamic system. This section examines effects of changes in some parameters. It is important to ask questions such as how a change in one country's conditions affects the national economy and global economies. First, we introduce a variable $\bar{\Delta} x(t)$ to stand for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

A rise in the total factor productivity of the HDE's tradable sector

We see what will happen to the global economy when $A_{1i}: 1.3 \Rightarrow 1.33$. The simulation result is plotted in Figure 2. As the system contains many variables and these variables are connected to each other in nonlinear relations, it is difficult to verbally explain these relations over time. As the total factor productivity is enhanced, the output of the HDE's tradable sector is enhanced, and the output levels of the other two economies' tradable sectors are reduced initially and are slightly affected in the long term. The global GDP and the GDP per capita of the HDE are increased, and the GDPs per capita of the other two economies are reduced initially and are affected slightly in the long term. Although the capital stock of the global economy falls initially, in the long term it is increased. The education and non-tradable prices in the HDE are increased, and the education and non-tradable prices are increased initially and affected slightly in the long term. As the HDE's tradable sector initially attracts more capital and labor, the other two sectors' output levels and human capital fall in the HDE and in the long term the other two sectors' output levels and human capital rise in the HDE.

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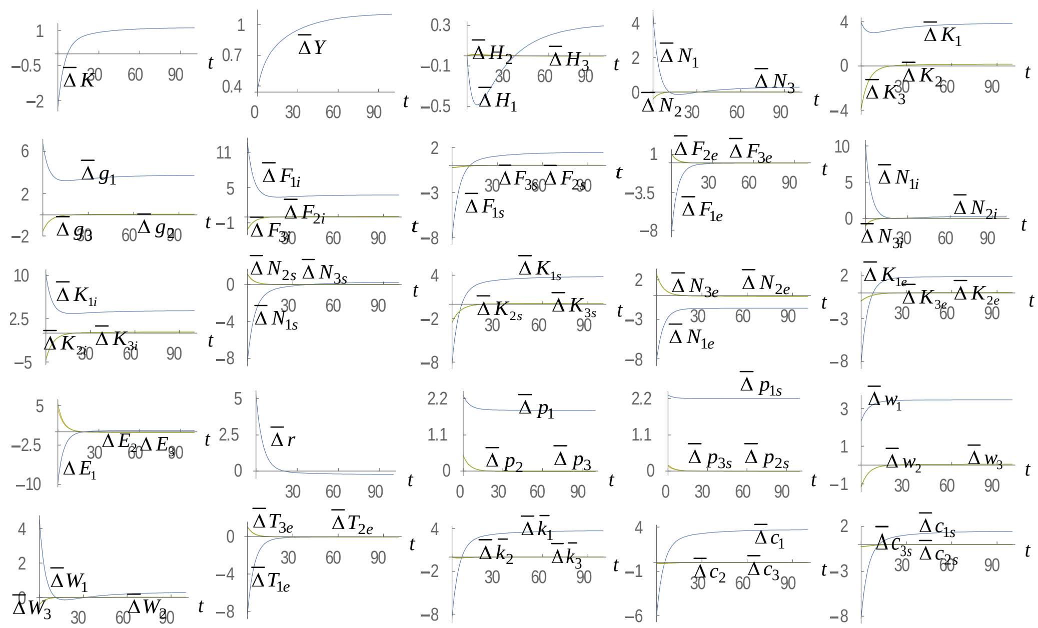

Figure 2 - A Rise in the Total Factor Productivity of the HDE's Tradable Sector

The HDE's propensity to receive education being enhanced

We now study effects of the following change in the HDE's propensity to receive education: $\eta_{10}:0.07 \Rightarrow 0.09$. The simulation result is plotted in Figure 3. The household spends more time on education. The HDE's education output is increased, but education price is slightly increased. The HDE devotes more resources to the education sector. The HDE's human capital is increased but not so much as the output and labor and capital inputs in terms of growth rates. The HDE's GDP per capita is increased, and the other two economies' GDPs per capita are slightly affected. Both the global wealth and GDP are reduced. The capital stocks employed by the three economies are reduced, even though the effects on the DE and LDE are very weak. Trade are affected. The HDE's trade balance is deteriorated, while the other two economies' trade balances are improved. From Figure 1, we see that more education does not improve global economic conditions. Although the HDE has higher human capital and study longer hours, the consumption levels and wealth per capita are reduced. It should be noted that this conclusion is based on the specified parameter values. We assumed that more education time does no improve human capital rapidly in the highly developed economy. By the way, in the model by Nakajima and Nakamura (2009), it is predicted that as the rich households demand higher education, the education price should be pushed up, the poor are excluded from higher education, and inequality between the rich and poor in the long term. Our international growth model does not have the same predictions if we consider the HDE and the LDE respectively as the rich and poor. As the HDE gets more education, the inequalities between the rich and poor with regard to income and wealth are reduced. Indeed, our model predicts different possibilities when the parameters are taken on different values. According to Barro (2013: 327), "Growth is insignificantly related to years of school attainment of females at the secondary and higher levels. This result suggests that highly educated women are not well utilized in the labor markets of many countries". Our simulation also suggests the possibility that growth may be insignificantly related to more education under certain circumstances.

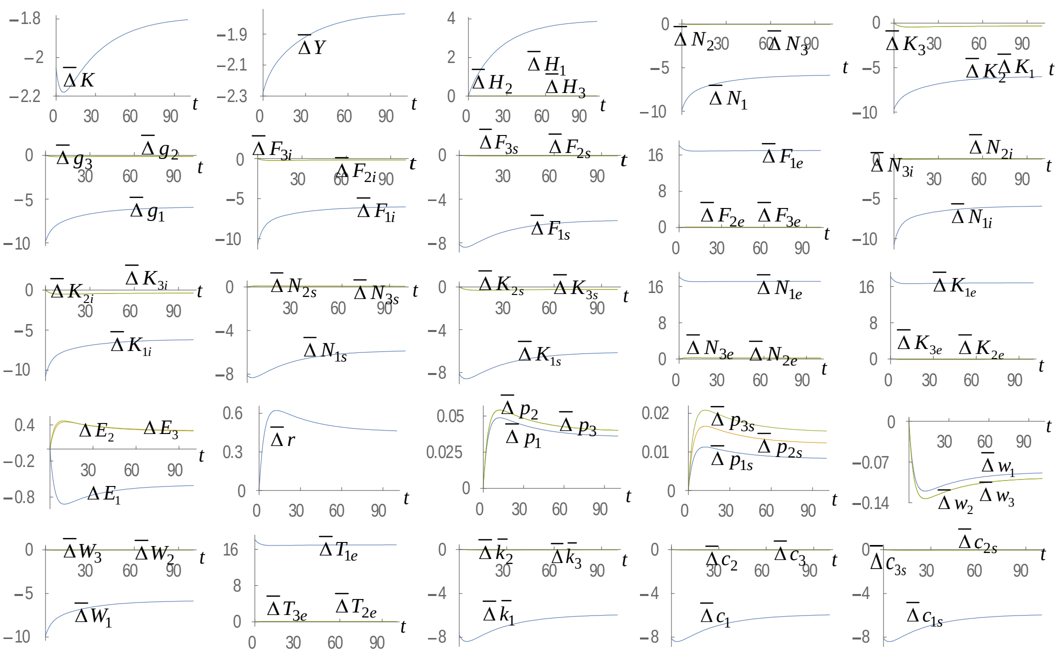

Figure 3 - The HDE's Propensity to Receive Education Being Enhanced

The HDE applying human capital more effectively

We now show effects of the following change in the HDE's human capital utilization efficiency: $m_1 : 0.9 \Rightarrow 0.91$. The simulation result is plotted in Figure 4. As the representative household applies human capital more effectively, the opportunity cost of education tends to be higher in the HDE. The representative household in the HDP spends less time on education. As the household works more effectively, the output levels of the tradable and non-tradable sectors and all the inputs of the two sectors are increased. Although the household spends less time in formal education, human capital is increased due to learning through doing and learning through consuming. The global wealth and GDP are enhanced. The wage rates of all the economies are increased in association with rising global wealth and falling rate of interest. The prices of education and non-tradable goods in all the three economies are reduced slightly. The HDE's wealth, consumption levels of the two goods are all increased. The output and labor input of the HDE's education sector are reduced. The sector's capital input falls initially and rises in the long term. This occurs due to interactions between attraction of capital by the other sectors and increase in the total capital.

The LDE learning more effectively in the education sector

We showed the effects of a rise in the HDE's human capital utilization efficiency. We now show what will happen to the global economy if the LDE learns more effectively through formal education in the following way: $v_{3e} : 1 \Rightarrow 1.1$. The simulation result is plotted in Figure 5. As the representative household learns more effectively, the human capital of the HDE is enhanced. The output levels of the tradable and non-tradable sectors are increased. The rise enhances the opportunity cost of education in the LDE. Initially the LDE reduces education time and scale of the education sector. As the LDE has higher human capital, the global wealth and GDP are increased. This brings down the wage rates in all the economies and brings up the rate of interest.

The prices of education and non-tradable goods are augmented. The DE and HDE spend more hours on education, even though the human capital levels are slightly affected. The HDE utilizes more capital and the DE and HDE use less. Our result also provides into the insight into what is observed by Barro (2013: 327-28): "Given the quality of education, as represented by the test scores, the quantity of schooling — measured by average years of attainment of adult males at the secondary and higher levels' is still positively related to subsequent growth. However, the effect of school quality is quantitatively much more important."

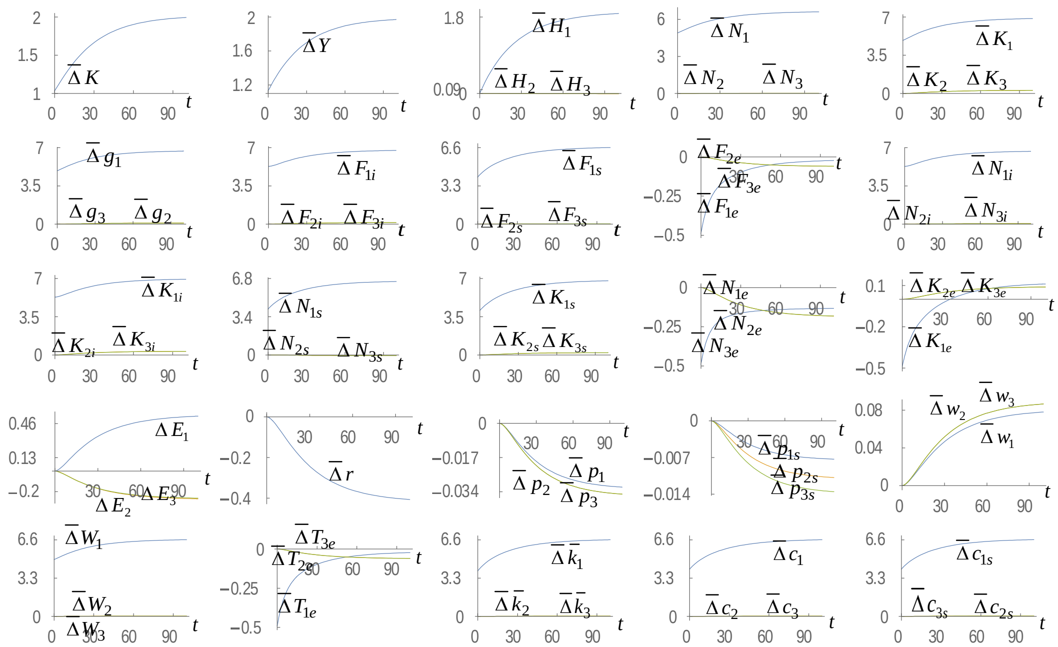

Figure 4 - The HDE Applying Human Capital More Effectively

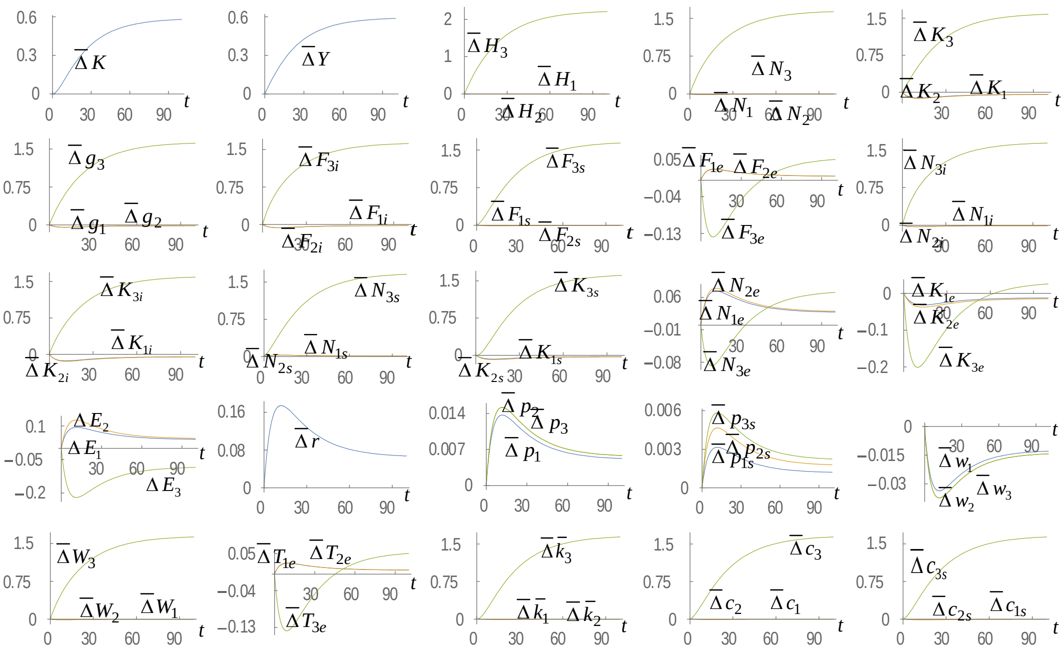

Figure 5 - TLDE Learning More Effectively through Education

The LDE preferring more domestic goods

We now study what will happen to the global economy when the LDE strengthens preference its domestic non-tradable product. We allow the propensity to consume the non-tradable good as follows: $\gamma_{03}: 0.06 \Rightarrow 0.07$. The simulation result is plotted in Figure 6. As the preference is changed, the representative household of the LDE reduces saving from the disposable income, consumes more non-tradable good, and consume less tradable good. As the demand for tradable goods fall in the LDE and the economy saves less, the global wealth and GDP are initially raised and soon reduced. The initial rise in the global economy is caused by the initial rises in human capital and initial low falls in the sectors' output levels. The falling capital is associated with rising in rate of interest and falling in the wage rates in all the economies. The HDE's household reduces education time and the other two economies raise education hours. The labor force of the HDE is increased and the other two economies' are reduced. The LDE's GDP per capita is increased and the other two economies' are reduced. The prices of tradable and non-tradable goods in all the economies are increased. The LDE's trade balance is deteriorated and the other two economies' trade balances are improved. All the economies employ less capital stocks in the long term. It should be remarked that Backus and Smith (1993) emphasize the importance of non-tradable goods as follows: We examine the possibility that nontraded goods may account for several striking features of international macroeconomic data: large, persistent deviations from purchasing power parity, small correlations of aggregate consumption fluctuations across countries …". As our approach deals with an integrated global market, our model is also able to provide insights into these issues.

The Economic Research Guardian – Vol. 4(2)2014
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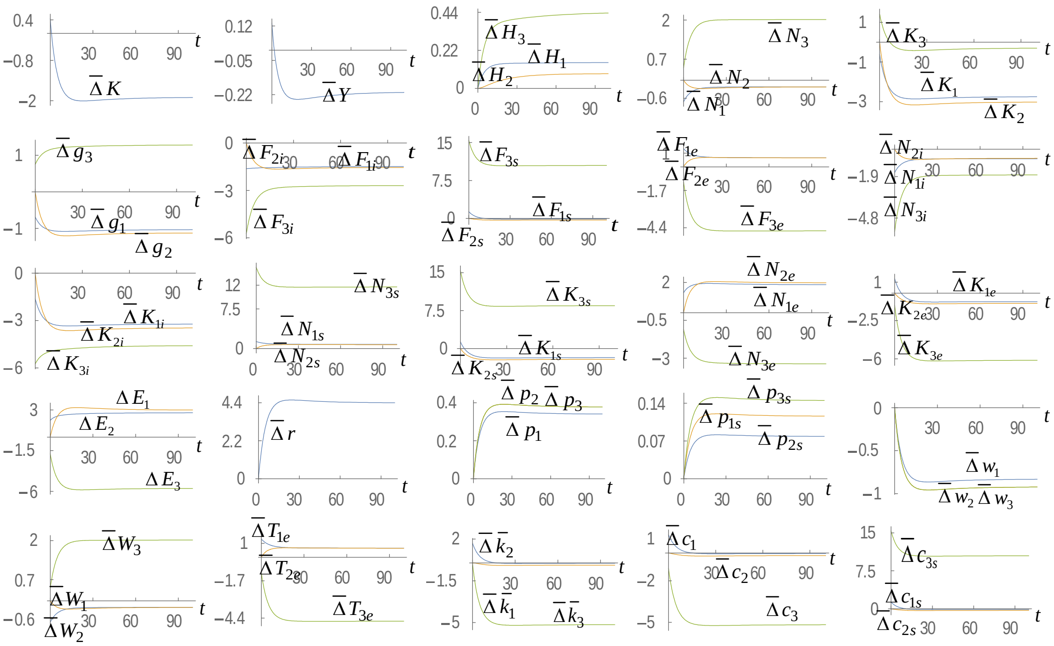

Figure 6 - The LDE Preferring More Domestic Goods

5. Conclusions

This paper built a multi-country growth model with endogenous study time, human capital dynamics and wealth accumulation under perfectly competitive markets and free trade. The model is built on the basis of the Solow-Uzawa neoclassical growth model with endogenous capital, the Uzawa-Lucas two-sector growth model with endogenous human capital, and the Oniki-Uzawa trade model. The model synthesized these well-known economic models with Zhang's utility function to determine household behavior. The model is for any number of national economies and each national economy consists of three sectors. Human capital is improved in three ways: Arrow's learning by doing, Uzawa's learning by education, and Zhang's learning by consuming. The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, and division of labor, and time distribution among education and work under perfect competition. We simulated the model for the economy with three countries to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in some parameters on the motion of the system. Our comparative dynamic analysis for three national economies provides some important insights into interaction between global economy and education. For instance, as the highly developed economy increases its propensity to receive education, the HDE spends more time on education, the HDE's education output is increased and devotes more resources to the education sector, the HDE's education fee and human capital are increased but not so much as the output and labor and capital inputs in terms of growth rates; the HDE's GDP per capita is increased, and the other two economies' GDPs per capita are slightly affected; both the global wealth and GDP are reduced; the capital stocks employed by the three economies are reduced; more education in the HDE does not improve global economic conditions. Another interesting case is about changes in the LDE's propensity to save. As the LDE increases the propensity to save, the LDE's wealth per capita is increased; the global capital and GDP are increased; The GDPs per capita are increased in all the economies; the HDE's human capital is enhanced and the DE's and LDE's human capital are lowered in the long term; the rise in the global capital brings down the rate of interest and raises the wage rates in all the economies; the prices of tradable and non-tradable goods in all the economies are reduced; and the HDE studies longer and the LDE and DE study less hours. There are also economic structural changes in all the economies. The total labor forces are increased in all the economies in the long term. The wealth and consumption levels in all the economies are increased, at least not reduced.

Acknowledgements

The author is grateful to the constructive comments of the anonymous referee and Editor Mihai Mutascu. The author is also grateful for the financial support from the Grants-in-Aid for Scientific Research (C), Project No. 25380246, Japan Society for the Promotion of Science.

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Appendix

We now derive dynamic equations for global economic growth. From equations (2), we have

$$k_{ji} = \alpha_{0j} \, k_{js}, \tag{A1}$$

where $\alpha_{0j} \equiv \beta_{js} \, \alpha_{ji} / \alpha_{js} \, \beta_{ji}$. From (4) and (5), we determine p_{js} by

$$p_{js} = \frac{\alpha_{0j}^{\alpha_{js}} \, \beta_{ji} \, A_{ji}}{\beta_{js} \, A_{js}} k_{ji}^{\alpha_{ji} - \alpha_{js}}. \tag{A2}$$

From the marginal conditions for capital in (4), we determine k_{ji} as unique function of k_{1i} as follows

$$k_{ji} = \left(\frac{\alpha_{ji} \, A_{ji}}{\alpha_{1i} \, A_{1i} \, k_{1i}^{-\beta_{1i}} - \delta_1 + \delta_k} \right)^{1/\beta_{ji}}, \quad j = 1, \ldots, J. \tag{A3}$$

From (4) and (7), we obtain

$$k_{je} = \alpha_j \, k_{ji}, \tag{A4}$$

where $\alpha_j \equiv \alpha_{je} \beta_{ji} / \alpha_{ji} \beta_{je}$ ($\neq 1$ assumed). From (A4), (4) and (7), we obtain

$$p_j = \frac{\alpha_{ji} \, A_{ji}}{\alpha_{je} \, A_{je}} \alpha_j^{\beta_j} \, k_{ji}^{\beta_j}, \tag{A5}$$

where $\beta_j \equiv \beta_{je} - \beta_{ji}$. From $w_j = \beta_{je} \, p_j \, F_{je} / N_{je}$ in (7) and (16), we have

$$n_{je} = \frac{p_{0j} \, T_{je}}{N_j}, \tag{A6}$$

where $p_{0j}(k_{1i}) \equiv \beta_{je} \, p_j \, \bar{N}_j / w_j$. From (A6) and (2), we solve

$$n_{ji} = \frac{(k_j - k_{js}) N_j + (k_{js} - k_{je}) p_{0j} \, T_{je}}{(k_{ji} - k_{js}) N_j}, \quad n_{js} = \frac{(k_{ji} - k_j) N_j + (k_{je} - k_{ji}) p_{0j} \, T_{je}}{(k_{ji} - k_{js}) N_j}. \tag{A7}$$

From (17) and (13), we have

$$n_{js} = \frac{\bar{N}_j \, \beta_{js} \, \gamma_j \, \bar{y}_j}{w_j \, N_j}, \tag{A8}$$

where we also use $w_j = \beta_{js} \, p_{js} \, F_{js} / N_{js}$. From (A7) and (A8), we solve

$$(k_{ji} - k_j) N_j + (k_{je} - k_{ji}) p_{0j} \, T_{je} = \frac{(k_{ji} - k_{js}) \bar{N}_j \, \beta_{js} \, \gamma_j \, \bar{y}_j}{w_j}. \tag{A9}$$

Insert $N_j = H_j^{m_j} \, T_j \, \bar{N}_j$ and $T_{je} = \eta_j \, \bar{y}_j / \bar{p}_j$ in (A9)

$$T_j = \frac{\hat{w}_j \, \bar{y}_j}{k_{ji} - k_j}, \tag{A10}$$

where

$$\hat{w}_j(k_{1i}, H_j) = \frac{1}{H_j^{m_j} \, \bar{N}_j} \left[\frac{(k_{ji} - k_{js}) \bar{N}_j \, \beta_{js} \, \gamma_j}{w_j} - \frac{(k_{je} - k_{ji}) p_{0j} \, \eta_j}{\bar{p}_j} \right].$$

From (9) and $\bar{p}_j T_{je} = \eta_j \bar{y}_j$ in (13), we have

$$T_j = T_0 - \tilde{w}_j \bar{y}_j, \tag{A11}$$

where $\tilde{w}_j \equiv \eta_j / \bar{p}_j$. From (A10) and (A11), we have

$$k_j = k_{ji} - \frac{\hat{w}_j \bar{y}_j}{T_0 - \tilde{w}_j \bar{y}_j}. \tag{A12}$$

From $K_j = k_j T_j H_j^{m_j} \bar{N}_j$, (A11) and (A12), we have

$$K_j = h_{j1} \bar{k}_j + h_{j2}, \tag{A13}$$

where we use $\bar{y}_j = (1+r)\bar{k}_j + T_0 \bar{w}_j$ and

$$h_{j1}(k_{1i}, H_j) \equiv -(1+r)(\tilde{w}_j k_{ji} + \hat{w}_j) H_j^{m_j} \bar{N}_j,$$

$$h_{j2}(k_{1i}, H_j) \equiv T_0 H_j^{m_j} \bar{N}_j k_{ji} - (\tilde{w}_j k_{ji} + \hat{w}_j) H_j^{m_j} \bar{N}_j T_0 \bar{w}_j.$$

Insert (A13) in (18)

$$\bar{k}_1 = \Lambda_k\left(k_{1i}, (H_j), \{\bar{k}_j\}\right) \equiv \frac{1}{h_{11} - \bar{N}_1} \sum_{j=2}^{J} (\bar{N}_j - h_{j1}) \bar{k}_j - \frac{1}{h_{11} - \bar{N}_1} \sum_{j=1}^{J} h_{j2}. \tag{A14}$$

where $(H_j) \equiv (H_1, \cdots, H_J)$ and $\{\bar{k}_j\} \equiv (\bar{k}_2, \cdots, \bar{k}_J)$. All the variables can be expressed as functions of k_{1i}, (H_j) and $\{\bar{k}_j\}$ by the procedure stated in the lemma. From the procedure and (15), we have

$$\dot{H}_j = \Lambda_j\left(k_{1i}, (H_j), \{\bar{k}_j\}\right). \tag{A15}$$

Substituting $\bar{y}_j = (1+r)\bar{k}_j + T_0 \bar{w}_j$ into $s_j = \lambda_j \bar{y}_j$ yields

$$s_j = (1+r)\lambda_j \bar{k}_j + \lambda_j T_0 \bar{w}_j. \tag{A16}$$

Substituting (A16) into (14), we have

$$\dot{\bar{k}}_1 = \lambda_1 T_0 \bar{w}_1 - R(k_{1i}, H_1)\bar{k}_1, \tag{A17}$$

$$\dot{\bar{k}}_j = \bar{\Lambda}_j\left(k_{1i}, (H_j), \{\bar{k}_j\}\right) \equiv \lambda_j T_0 \bar{w}_j - (1 - \lambda_j - \lambda_j r)\bar{k}_j, \quad j = 2, \ldots, J, \tag{A18}$$

in which $R(k_{1i}, H_1) \equiv 1 - \lambda_1 - \lambda_1 r$. Taking derivatives of equation (A14) with respect to t yields

$$\dot{\bar{k}}_1 = \frac{\partial \Lambda_k}{\partial k_{1i}} \dot{k}_{1i} + \sum_{j=1}^{J} \Lambda_j \frac{\partial \Lambda_k}{\partial H_j} + \sum_{j=2}^{J} \bar{\Lambda}_j \frac{\partial \Lambda_k}{\partial \bar{k}_j}, \tag{A19}$$

where we use (A15) and (A18). Equaling the right-hand sizes of equations (A19) and (A17), we get

$$\dot{k}_{1i} = \bar{\Lambda}_1\left(k_{1i}, (H_j), \{\bar{k}_j\}\right) \equiv \left[\lambda_1 T_0 \bar{w}_1 - R\Lambda_k - \sum_{j=1}^{J} \Lambda_j \frac{\partial \Lambda_k}{\partial H_j} - \sum_{j=2}^{J} \bar{\Lambda}_j \frac{\partial \Lambda_k}{\partial \bar{k}_j}\right] \left(\frac{\partial \Lambda_k}{\partial k_{1i}}\right)^{-1}. \tag{A20}$$

In summary, we proved Lemma.