

ON THE CONVERGENCE AMONG THE EU FIVE MAIN COUNTRIES IN A CLUB MODEL UNDER A MEMORY HYPOTHESIS WITH SPREAD MEASURE

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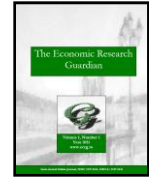
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Abstract

After an examination of the public choice theory of clubs and of Buchanan theory regarding Governments as club and after the analysis of the theory of convergence for the optimal currency area theory in club perspective, the paper focuses on the EU (European Union) and EMU (Euro-monetary Union) as clubs. On the basis of the modern growth theory, we develop an empirical research on the convergence path, in a new model, under the memory formalism hypothesis, with spread measures, on the 5 EU main countries, with 15 parameters deducted from neoclassical theory of growth and from the EU-EMU rules. The convergence and stability of the 5 main countries in the EU club, France, Germany, Italy, Spain and UK as affected by the EMU club(to which 4 of the 5 countries do belong), is examined, by measuring their 15 teen parametric spreads, from 2003 (first year of normal circulation of Euro) to 2011 (last year of available official statistics). The convergence with growth actually developed before the great financial fluctuation, in which divergence spread out. Convergence, then, reappeared, but did not last long, because the consolidation policies were not counteracted by appropriate expansionary fiscal policies at the EU level and by timely non conventional expansionary policy. Divergence cum semi stagnation did develop. GDP and unemployment are the dominant parameters, while GDP per capita is the least important. A memory explanation of the paradox is suggested. To assure stability cum growth of the two clubs, one needs monetary and fiscal policy tools coherent with their models, which are already institutionally available. Further integration would face the same issues, in a less free situation.

Keywords: Convergence, Club model, Memory hypothesis, EU

JEL classification: D71



1. The governments and the governments' union as clubs

1.1. The club approach in the public goods theory and the Governments as clubs

Our paper studies the issue of the viability of the club approach for the main EU and European Monetary Union member states. By “viability”, we mean the “survival capability” of the monetary union of the euro zone, *with all of its member or with most of them*. Club goods, according to the familiar definition of James Buchanan [Buchanan (1965 and 2001) and Buchanan and Goetz (1972)]¹ are an intermediate category of public goods that are common in their use, but to some extent, excludable for those who do not accept to enter in a “Club” that offers them, in competition with other “Clubs”. The institutions offering club goods may be public or private; in both cases, they must be non-profit-institutions of common use of the given good. Buchanan defines this requisite as “non-ownership”, meaning that the subject operating the good is an institution devoted to the interest of all users. One could say that this subject, which Buchanan defines it as a club, is a “common” or a “cooperative”, a “condominium” or an “association”, depending on the activity, the rules of decision making and the type of property right, in the broad sense of this term. The decision-making may be one subject - one vote - or one subject according to his share of right. The type of “property right” indicates the owner of the real property, the right of use, or a mix of them etc. However, many common goods, belong to profit-firms.

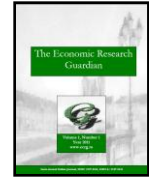
In the pure model the Clubs do not have a territorial jurisdiction on their members. The members do not have a “residence” in the space occupied by the pure Club territory. They have a residence either in a territory where the pure club is located, in which is normally the case for the school of compulsory or optional education as a club², or elsewhere in the same region or state, or outside of them. The last case may be usual for a toll road and a lighthouse³. Therefore, the mobility from one pure club to another, does not imply the loss of local rents, for those who leave it. Those who enter in the “club” have the choice of opting out, in a setting of perfect mobility and of no exploitation by the owner of the good. The non-territorial club is similar to a taxi or a rented car: one takes it, pays for the services and sorts out at the end of the trip or, if he wants, even before⁴. The common usage may imply positive and negative effects of the presence and the use of other members, i.e. external economies and diseconomies from the other users. In some case, the most important service of the club consists in putting together the different members and their additional services, merely aiming to ease their exchanges. In the case of multiple members, some of them may harm other members with their presence or behavior, thus the potential members must weigh these harms with the benefits of the presence and behavior of others. In the case of a multiple common goods club, supplying goods with different benefits and damages, according to the preferences of the potential

¹The theory of club goods has had a broad theoretical development and diverse application [see Pauly (1970a) and (Pauly 1970b), Berglas (1976), Sandler & Tishart (1980), Breannan & Flowers (1980), Casella & Frey (1992), Cornes & Sandler (1996), Sandler & Tishart (1997), (S.Scotchmer 2002)] .

² Recall that the school is a pure club only if is not a profit-oriented.

³ Notice that the toll road and the lighthouse are clubs only if is a non-profit institution either public or private, as happens in most case.

⁴ However, normally, the subject that owns the taxi or the car rent is not a club of those who use it.



participants, the positive and negative benefits of each of the goods, and of those jointly supplied, should also be weighted. Indeed while some good are beneficial, others may be damaging.

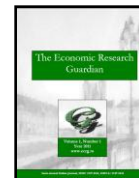
The potential members shall enter in one of these clubs only if the benefits from the use of the goods which give more benefits than costs, are higher than the use of the goods with costs that exceed the net benefits and the costs of the presence of other members. In case of too heterogeneous preference of the potential entrants, a club might adjust the rules by smoothing the type of members and conducting the members and goods supplied, in order to distribute the welfare losses among all members, in an attempt to minimize them⁵. In a large-club with heterogeneous preferences, this minimization may be too difficult and costly. Therefore, rational behaving clubs do not try to be too wide, even if this could allow economies of scale reducing their unit costs. Anyway, the clubs of public goods that one may observe (and the private clubs too) normally imply a relevant degree of homogeneity. Their reason is that those who have dis-homogeneous preferences do not enter in them or, once entered, leave them. In the case of common goods, necessary for the ordinary life and business, if private clubs are impossible, forbidden or inadequate, a government has to take care of them. And here arises the issue of the Governments as territorial clubs, first theorized by Buchanan and Goetz (1972), as for the Local Governments, in which they analyze the sub-optimality of the Tiebout model of competition among Governments having the same kind of jurisdiction on different sections of the territory of a given state or union or states⁶. They imply a locational dimension, with territorial rents. As the civil society cannot exist without Government with territorial jurisdiction of law and order and other basic common goods, any person, either individual or collective, needs to live under one or more Government, with their own territorial jurisdiction.

According to Buchanan and Goetz (1971) and Buchanan (1997) these Governments are territorial clubs as far as they observe the principle of non-ownership and allow the migration to other similar public good clubs, in a free choices setting. There might be some alternatives in the private sector⁷. They can increase, but in real life Governments bureaucracies and politicians resist to their extension, because it reduces their power and control. The high mobility assumption that solves the homogeneity issue in the non-territorial clubs is not easily applicable to the territorial clubs. The migration to another territory, ruled by another municipality implies the loss of the location rent and

⁵ See Fedeli and Forte (2002), with Comment by Chakrawarty and Skott (2002).

⁶ See Tiebout, C. (1956), *A Pure Theory of Local Expenditures*, journal of Political Economy,): 416-4.

⁷ The private schools of compulsory education and private hospitals in countries where the lower level governments supply a public service of this kind may be an example. The schools of non-compulsory education may behave as a-territorial clubs offering jointly different disciplines, for some week or month or for one or more years. Those who consider whether to attend to them, before making the choice, shall weight the benefits and costs of the joint disciplines. If they feel that may not get an *aggregate* benefit lower than the cost in terms of money, time and efforts employed, shall better try other schools. Once have made the choice, if they are unsatisfied, they can choose another school, before the completion of the courses. The cost of leaving consists in the waste of knowledge caused by the interruption of the learning process. On the gain side, they consider the alternative schools and the saving of costs of not going to any school. If education is compulsory up to a given age, the opting out shall be feasible only if there are relevant alternatives as clubs or firms offering that kind education. The scholastic institution has power of ruling and of supplying services only on its education matters, under the Governments' regulation. In all the other matters, the school, its personnel, its students, must follow the rules and consume the services (if any) provided by the Governments that have jurisdiction for the law and order on that territory.



may additionally imply the loss of common ownership, because it may be possible that the other municipalities do not apply to the immigrants the equal right principle of the club model.⁸ However, the cost of opting may be reduced and the freedom of choices broadened if there are higher-level governments that assure the fruition of many not (entirely) decentralized public goods.⁹ . On the other hand, the concentration of the fiscal power in the central level may reopen the non-ownership issue. As seen, the possibility of opting out is the only mean to assure the non-ownership principle. Therefore, even at the upper level of the territorial clubs it is desirable to have a multiplicity of public entities assuring important common goods in competition among them. One of them is a “good money”.

1.2. The conditions for an optimal monetary area and for a monetary union as “a club”

Money, as a common good in theory infinitely non-rival is a bundle of joint goods because it is both a medium of exchange and a store of values, for both the transactions in the same money and for those in other money, in the real sector and in the capital sector. The main requisite of money is “monetary stability”. But this is an inherently ambiguous principle. It means that the Central Bank cannot take in consideration the Philips curve in which the level of employment depends on the level of prices: their increase raises the employment level, by increasing the level of prices to devalue the nominal wages. The central bank cannot finance the deficits of the member countries by buying its new public debt nor monetize the high public debts *by an inflationary policy*. This principle, may symmetrically imply that the central bank could buy public debt to avoid a deflation, i.e. spiraling reduction of prices that endangers the monetary stability. An exogenous devaluation of a main foreign currency may imply a revaluation of the domestic currency, independently from a criterion of purchasing power parity. Is this an infringement or not of the monetary stability criterion? The different interpretations of the monetary stability principle may lead to different monetary policies.

Countries with higher nominal deficits and public debts ratios to GDP may prefer a higher price level than countries with low deficits and low debt/GDP ratios. Less competitive countries may prefer a lower rate of exchange than countries with high rates of exchange. On the other hand, money is a “necessary common good” because without it, just as without a language, one cannot enter in relation with the other members of a civilized society. A Monetary Union among different countries may be, thus, a result of the market enlargement . However, to be successfully, a unique market is not enough, as the preferences for the monetary policy must be homogenous. To have a well-functioning monetary union, i.e an optimal currency union, the realization of the conditions of an optimal currency area is not sufficient.

On the other hand, the “monetarist” theory of the optimal currency area (Mundell 1961 and 1963) sets necessary but not sufficient conditions for the unique market. According to this theory, no barriers should exist in the mobility of the production or the final goods factors. The perfect mobility of the production factors would insure that the central bank interest rate does not originate

⁸ See Buchanan ad Goetz (1971) in Buchanan(2001),§ 5 “The reality of Fiscal Mobility”, p.61.

⁹ See Buchanan ad Goetz (1971) in Buchanan(2001),§ 6.Conclusions.



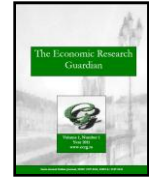
different interest rates in the different countries. The differences in productivity between various areas, a given exchange rate may create current account surpluses in the balance of payment for some member countries and deficits for others. On the other hand, the homogeneity of productivity tends to increase within the monetary union, once created, through the mobility of the production factors .

The migration of the capital (and firms) to the areas in which their productivity considering the level of the wages, is higher. Labor may be more sticky. If the rate of exchange is flexible, its devaluation, reducing the real value of wages could solve the problem of the low productivity areas, caused by nominal wages that are too high. With rigid labor contracts and rigid rates of exchange unemployment shall increase. Sooner or later, the workers of the countries with “wrong” nominal wages shall understand that the rate of exchange devaluation is a mere monetary illusion for the devaluation of wages. Therefore, they shall opt for the flexibility of wages in order to cope with the differential in productivity [Swoboda (1999)], thus making possible the entrance of their country in the monetary union, and to get the benefits of the enlarged market. If a country has adhered to a monetary union, without the reform of the labor market, the money illusion that a devaluation may solve the problems of disequilibria may remain, in the labor unions of the low productivity countries, accustomed to periodic devaluations of their currencies to solve the problems of the balance of payment disequilibria. However, maintaining the constraint of the fixed rate of exchange shall oblige the unions to avoid the request of increased wage rates because they might create unemployment (Mundell 1973). More generally, the constraint of the fixed rate of exchange shall induce the unions to accept wage flexibility (Mc Kinnon ,1999). However, why labor union would adopt this line instead of asking the government to take care of the unemployed? The constraint of the fixed rate of exchange may not work. The mobility of the factors of production may be imperfect because of the existence of local and regional rents. Therefore, the problem of homogeneity of the preferences needed for an optimal monetary union, as a well-functioning club, remains¹⁰.

The first requisite for a public institution to be a club is the non-ownership principle that implies that the decision making rule and the activity must be informed to the proportionality principle, which is actually adopted for the MU both as for the decisions of the ECB and for those of the EU, to which EMU belongs. However, as for the second requisite of the club, that of freedom of exit, in a regime of conventional money as the present one, a double paradox arises. First of all, unlike a

¹⁰ The EU-Maastricht rules and the Stability and growth pact do not seem to have considered the theme of the requisites for the optimal currency area, as have merely focused on parameters of monetary stability parameters. Likely they have been considered as constraint to undertake structural reforms The condition to enter in Emu area:1 inflation rate of a given Member state must not exceed by more than 1.5 point that of the three best performing Member states in terms of price stability; .2 The annual government deficit must not exceed 3% of GDP 3. Government debt must not exceed 60% of GDP and those who exceed it should be reduced.4. Exchange rate: Applicant countries must not devalue their currency. Member state must have participated in the exchange-rate mechanism under the European Monetary System (EMS) for two consecutive years before the examination, without severe tensions.5. Long-term interest rates must not be more than 2% higher as those of the three best performing Member states in terms of price stability.

The fiscal compact has introduced two new stringent fiscal parameters of monetary stability, i.e. the principle of budget balance for the general government budget corrected for the cycle and a yearly reduction of the deficit/Pil ratio exceeding 60% of GDP.



money endowed of intrinsic value, the conventional money cannot be a pure club good, it must be a territorial good, because it is a mere credit note with legal course on a given territory. To assure its circulation and its value, a Central Bank with monetary jurisdiction is needed on that territory and, possibly, with a control on its financial intermediaries system. Thus, at best, a common money a territorial club good of a club of states, forming a monetary union¹¹. On the other hand, in order to insure the credibility of a given conventional money one must assure the credibility of the state or the union of states where it is legally circulating. If this state or union of states risks to dissolve, the conventional money could also lose value and even become less valuable. Therefore it is in the strict interest of the monetary union to hinder the exit of a member state, particularly if it is an important one, in order to oblige it to share the aid cost and to constrain its behavior to make it consistent with the club's membership.¹² The Monetary Union must be very careful in accepting as members only homogeneous states with a stable and a common interest to their participation. In addition, it should also have an interest in behaving to assure this homogeneity.

The Union of States of which the Monetary Union is member too has an interest in adopting the adequate instruments to that effect, because of the effects of the Monetary Union on the convergence of its members.

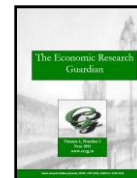
To sum up, the governments of the countries that do not converge to the virtuous path, are obliged to stay and to try to conform to the rules of EU and of EMU. The situation for the non-performing countries as for the choice of the membership to EMU is similar to that of the contract of Faust with the devil. The first step is voluntary, the further are obligatory. The countries over fitted might instead decide to leave the club, violating the irreversibility pact. All this implies an adaptation of its less fitted member states to the convergence to conditions in which they can survive and develop and the choice of remaining outside if they are not strong enough to converge.

This is the main reason of the focus of the present research on the five main members of the EU, in which four belong to EMU, while one is member of EU, but may sort out if EMU breaks down.

The reminder of the paper is as follows. Section 2 describes the model. Section 3 presents the data and the empirical findings. The last section concludes.

¹¹ For EU and EMU as club see T. Mora (2006) and Wholgemuth and Brandi (2007) and Wolgemuth (2011) who defines EU as a multiple options club of clubs one with EMU and other without.

¹² One should not confuse the Monetary Union with the currency association of a state to the currency of another state. Argentina pegged its pesos to the US dollar. The pegging did not work and Argentina was obliged to leave the legal parity with the dollar. Lichtenstein has pegged its currency to the Swiss franc with better results. The Kingdom of Monaco, the Republic of San Marino and the Vatican State use the euro, as their currency by a bilateral pact with the EMU i.e. ECB. If they leave the Euro breaking the pact, they must wither issue their own currency or, more easily, adopt another currency, by a bilateral pact. The two examples of the past of monetary unions – the Latin Monetary Union and the Scandinavian Monetary Unions, both of the XIX century do not correspond to the territorial club model of EMU, as there was no central bank.



2. Convergence and club homogeneity. A model with memory formalism

2.1. Convergence in endogenous and exogenous growth models

“Clubs convergence” – first employed by W. Baumol (1986) - has several different meanings¹³. Some derive from the ambiguity of the subjective component of the paradigm, i.e. notion of club, referred to an institutional fact or to a statistical situation.¹⁴ Others relate to the object of the convergence, which differs according to the different point of view from which is studied. In this case, we are considering the convergence of national governments belonging to a given club of governments, conceived as an organized public entity, to a situation of homogeneous parameters from the point of view of a growth-cum employment fiscal and monetary policy of the economic and monetary policy of the club.

The concepts and formulas of convergence in relation to growth, have traditionally been dealt with the analysis of the behavior of the neoclassical exogenous models of economic growth, respectively of Solow and of Sala y Martin.

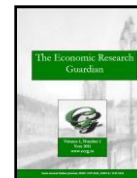
In Solow ‘model [Solow (1956), Mathunjwa and Temple (2007) and Barro & Sala Y Martin (1994), Chapter 1] with only one final commodity representing GDP under a fixed rate of saving and a correspondent investment in capital K, the GDP growth is determined by “k “under a diminishing return hypothesis and by the exogenous technological progress. Under the given technological progress and k/GDP, the “high GDP “countries H have a slower rate of growth than the smaller GDP emerging countries E, that started growing more recently, ensuring a convergence path [Islam (1995), Caselli, Esquivel and Lefort (1996), Bernanke and Gurkaynak (2001), Masanjala and Papageorgiou (2004) and Beaudry, Collard and Green (2005)]¹⁵.

As argued by Barro and Sala y Martin (1991 and 1994) and by Sala Y Martin (1996), in the lower income countries the Endogenous growth due to gradual diffusion of technology may give origin to a sustained growth .Labor productivity may increase, under flexible labor supply and increased level of skills. The marginal price of labor is lower E countries, because a lower average income also implies low average wages of goods and personal service prices. In a single market, the firms of H will decentralize their production in E because even if their productivity reduces, it cannot be lower if they adopt there their technologies. Therefore, in a simplified model of full employment without output gaps, like the ones considered, an economic and monetary Union may enhance growth.

¹³ See Dorwick & Duc Tho Nguyen (1989), Barro & Sala y Martin (1992), Barro & Sala y Martin (1994), Galor (1996), Ben-Daviv (1996),Evan & Carra (1997), Ben-David (1997),Ortiguera &Santos, (1997), Reiss (2000), Dowrick &A.J.B De Long (2003),Islam (2003),Lee &McAleer (2004), Busetti, Forni, Harvey, &Venditti (2006), Gil-Alana& Perez de Gracia (2006) , Cunado, Gil-Alana &Perez de Gracia (2006), Fischer and Stirbook (2006)Mathunjwa and Temple (2007) Cavenaille and Dubois (2010) , Caputo (2012).

¹⁴The proper concept of club refers to the club as an institution. The figurative concepts refers to the club as a statistical notion of cohort of subjects in which one finds the ones statistically similar for given characters, i.e. the “clusters”.

¹⁵ The “Global Competitiveness” model of the World Economy Forum.



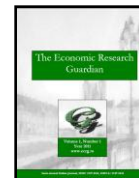
However, even in these models, without “output gaps”, divergence may reappear inside the H and E countries and between them. Young skilled labor may try to migrate to the best performing countries, if it cannot find a suitable employment in the home country, and therefore it does not lose a relevant location rent. The firms that externalize their production in other states of the club or elsewhere became more competitive also in the H countries. Organizational process underlying growth, government policies, industry clusters, market organization, civil relationships, *etc.* have varying permutations and strike differently according to culture, history and political system. Memory may be a relevant factor in the convergence process (Caputo 2012).¹⁶ The “Global Competitiveness” designed by Xavier Sala Y Martin introduces political economy factors of growth. It considers “macroeconomic requisites” as monetary stability, given by a low but positive inflation rate and fiscal soundness defined as small public deficit/GDP and moderate debt/GDP. These factors, together with the size of the tax burden, may also be relevant from the point of view of the homogeneity parameters of EU and EMU.

2.2. `15` Parameters of CLUB homogeneity to assess its convergence and viability hypothesis

We therefore consider the following 15 parameters as parameters relevant for measuring the convergence in EU and EMU as clubs, in a club of clubs system, as for the 5 main countries, one without EMU and 4 with EMU.

Table 1 - Parameters of CLUB homogeneity to assess its convergence and viability hypothesis

1	growth GDP
2	GDP per capita
3	unemployed/GDP
4	Labour product per person
5	labour product per hour
6	investments/GDP
7	Gross savings/GDP
8	VA agriculture/GDP
9	VA industry/GDP
10	inflation rate
11	public expenditure/GDP
12	general government deficit/GDP
13	balance payments
14	balance payments current
15	bond yield



The first 7 are the parameters of the main neoclassical growth model. The next 2 are a specification of the 7 parameters relevant for the EU policy and the other 6 are the financial parameters of the Maastricht Treaty and of the fiscal compact, that condition the monetary and fiscal policy of EU and EMU as clubs of Government.

We consider the 5 most important economies of the EU, namely France, Germany, Italy, Spain, UK. We consider the spreads of their 15 parameters of convergence in the period 2003-2011, which is actually short relative to those of the usual economic evolution, but 2001 is the first year of EMU and the first two years were years with peculiar perturbations due to the transition to the new monetary system. We begin from the moment when the new system was set at regime, in a club model. Our attempt is to also verify if the members are actually converging in spite of the crisis occurring in the period considered from 2007-8 on.

2.3. The convergence model with memory formalism for the members of the club of countries in an Economic Union

In a previous paper it was considered that a club is formed by n members $y_i(t)$ and assumed that the members are related to each other by the equations

$$D^\nu y_i(t) = \sum_1^m a_{ij} [y_j(t) - y_i(t)] \quad (1)$$

$$\text{where } D^\nu(t) = (1/\Gamma(1-\nu)) \int_0^t f'(u) du / (t-u)^\nu \quad \nu \in]0,1[\quad i=1,2,\dots,n \quad (2)$$

which, since it reproduces the ordinary derivative when ν is positive integer, is called, perhaps improperly, fractional derivative of order $\nu \in]0,1[$, $\Gamma(\nu)$ is the gamma function. The definition (1) is found in many treatises (e.g. Caputo 1969, Podlubny 1999, Kilbas and Marzan 2005, Magin 2006, Mainardi 2010, Diethelm 2010). In practice, the derivative of fractional order of $f(t)$ is constructed by taking a weighted mean of the first order derivative $[df(t)/dt]_z$ in the time interval $[0,t]$, so as to induce a sort of feedback system. That is the values of $[df(t)/dt]_z$ at time z far apart from t are given smaller weight than those at times z closer to t . Given that the weights are increasingly smaller with increasing time separation from time t , the effect of the past fades away as time goes by. When $\nu = 0$ and $f(0) = 0$, the fractional order derivative reduces to the function itself.

In our case the memory formalism represents the effects of the previous gradients to the present value of the economic parameter considered; moreover, it would be the past influencing the present of the parameter which seems more than reasonable in agreement with the statements of Demaria (1978) and of Galbraith (1972) concerning the evolution of economy and its models.

A possible interpretation of the mathematical memory based model of this note is that it may indirectly represent the existing institutions and procedures, causing inefficiency and internal rigidities as delaying factors in the evolution of the economy. This eventually takes also into account,



in a somehow abstract form, that the second principle of thermodynamics, which takes all systems of nature to the same energy level by energy dispersal (as suggested by Annala and Salthe 2009). However, a more comprehensive interpretation in the very complex field of economy evolution is desirable.

Equations (1) implicitly assumed that there is interaction between the club members and that exogenous forces are acting on each member of economy. Equation (1) implies that eventually the club members will converge to a common condition. We finally note that formalism of the system (1) mimics Fourier equation.

2.4. The convergence of France, Germany, Italy, Spain, UK. in the EU club by measuring their distances

The evolution of the club will be studied using the distances between the club members which,¹⁷ [...] Our work¹⁸ is based on the following hypotheses

- a) *Larger spread of distance between the members of a clubs means large inhomogenities.*
- b) *Large spreads between two or more members is a sign of instability of the club.*
- c) *The inhomogeneity is measured with a single parameter U defined in Table 1.*

Let m be the number of economies in the club and n the number of parameters, in our case then $m = 5$ and $n = 15$. The spreads of the distances are obtained by normalising each parameter p_j to the maximum value of its norm, acquiring a new set of normalised parameters q_j and considering the set $x_{i,k,j}$ of the couples of difference of the normalised parameters q_j ; p_j is then substituted with

$$q_j = p_j / |p_{j\max}| \quad (3)$$

where $|q_j| \leq 1$ defines a new Cartesian space.

We first assume the case when all parameters p_j have positive values and consider the differences

$$x_{i,k,j} = q_{ij} - q_{kj} \quad (4)$$

¹⁷ This has already been done Caputo and Kolari (1990, 1997) for banks identified through their indicators as Cartesian coordinates and the use of the Hamming algorithm as a check of the results of the application of the pattern recognition method itself

¹⁸ The evolution of the 5 Club members considered here has already been tentatively studied using 29 parameters taken almost at random among those available in the 3 years 2000, 2005, 2010 (Caputo 2014); but the results, due to the limited resolution and the limited time interval used, were inconclusive.



with $|x_{ik,j}| \leq 1$ which are the components of an abstract distance between the economy identified by k from that identified by i relative to the parameter j in the Cartesian space of the parameters q_j . From the definition (3) follows that

$$\sum_{j=1}^n [(p_{ij} - p_{kj}) / p_{j\max}]^2 < n \quad (5)$$

or

$$D_{ik} = [\sum_{j=1}^n [(q_{ij} - q_{kj})]^2]^{0.5} / \sqrt{n} < 1 \quad (6)$$

where D_{ik} is the abstract distance of the economies i and k . in the Cartesian space defined by the parameters q_j .

The normalizing factor of D_{ij} is obtained first considering the case when all parameters assume non negative values and that m is even: consider now that if the values of the parameters of a given subset of $u < m$ of the m economies of the set are unity and all the others are zero, then the sum of all the $m(m-1)/2$ distances is $n^{0.5}u(m-u)$ whose maximum is obtained when $u = m/2$ which gives the distance $m^2 n^{0.5} / 4$. If one, or more than one, of the zero value parameters were to assume a positive value the sum of the distances would decrease. The same applies to the case when the values 1 are smaller than 1. The case when m is odd is obtained with the same procedure.

It is seen that when all parameters assume non negative values, the sum of the distances D_{ik} is smaller than

$$n^{0.5} m^2 / 4, \text{ when } m \text{ is even} \quad (7)$$

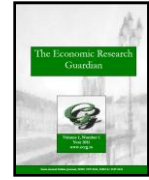
$$n^{0.5} (m^2 - 1) / 4, \text{ when } m \text{ is odd}$$

which we, for simplicity, assume as normalizing factor of the distances.

Finally, taking into account the possible presence r parameters which may assume negative values and that the corresponding values of $x_{ik,j}$ are subject to the limit $|x_{ik,j}| \leq 2$, formula (7) are approximated with

$$U = D_{ik} / [(n + 3r)^{0.5} m^2 / 4], \text{ when } m \text{ is even}$$

$$U = D_{ik} / [(n + 3r)^{0.5} (m^2 - 1) / 4], \text{ when } m \text{ is odd} \quad (8)$$



where the spread of the values of U could tentatively be considered as the abstract measure of the inhomogeneity of the set of economies. The homogeneity of the set is then inversely proportional to the value of U .

Obviously the distances obtained are only abstract indicators of the homogeneity degree of the different economies, member of the club, with the understanding that larger values of U imply relevant differences in them.

It is essential to underline that the value of U of the Club is of lesser meaning. Their spread within the club is of greater importance as a direct measure of the club inhomogeneity and a signal of instability. The meaning of U would specially emerge in the comparison of its values obtained at subsequent times; these values may indicate whether the members of the set of economies is becoming less or more homogeneous, that is, to converge to a unique state where all the parameters are theoretically equal.

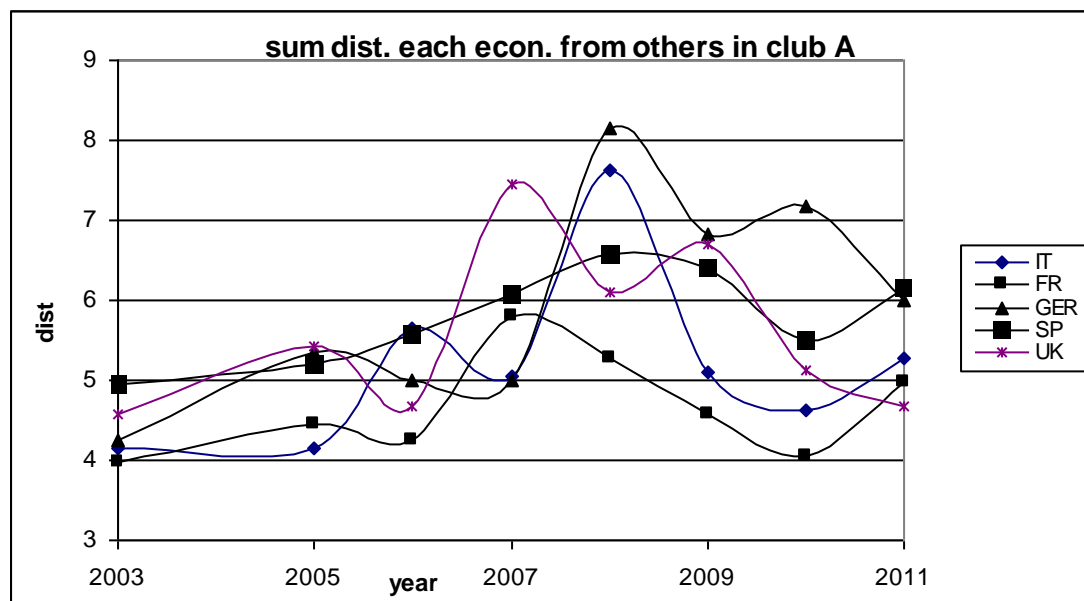
2.5. The measure of the stability of the EU club, formed by France, Germany, Italy, Spain, UK, from 2003 to 2011

In order to study the stability of the club we considered the values of the 15 parameters in the 8 years 2003, 2005, 2006, 2007, 2008, 2009, 2010, 2011, covering the presumed stability before the crisis began in 2007.

We note in Figure 1 that the maximum rate of $U_i(t)$ occurs in the years 2007 and 2008 after a rapid increase from the preceding value implying a rapid increase of inhomogeneity and possibly of instability; the increase is two folds significant: in the amplitude and in the rate.

Concerning the stability, we note in Figure 1 the large rapid oscillations of $U_i(t)$, which imply significant instability of the club. We also note the significant sharp increase of $U_i(t)$ before 2008, just one year before the down turn of the economic cycle and subsequent crisis.

In order to see which member of the club gave a relatively larger contribution to the instability, we consider the Figure 1a where we note that



Note: Values of $U_i(t)$ for each club member.

Figure 1 - Values of Spread $U_i(t)$ for each club member

Italy, Germany and the UK recorded the largest value of $U_i(t)$ and of the rate of change of $U_i(t)$ which is significant for their inhomogeneities and instability as well as for those of the club. The instability of the club in the period 2007-2010 is evident. The stability and convergence seems to reappear in 2011, but it is not certain.

As for the stability and convergence, it is worth noting that in this period, they are reached without GDP growth.

It is worth noting that the instability within a *single* club member (that is larger values of $U_i(t)$) may be due to reforms that results in a better performance of the economy.

In Table 2 we report the values of $U_i(t)$ in the 8 years period for each member of the club. The largest variations of inhomogeneity occurred between the years 2007 and 2008 in Germany (8.14-5.0) and Italy (7.63-5.05) and between the years 2006 and 2007 in the UK (7.44-4.68). They may derive from both, the different impact of the crisis on countries with less or more elastic structures, but also from reforms undertaken in order to take care of the structural inadequacies of the economy or/and by the lack of them in one of the considered countries.

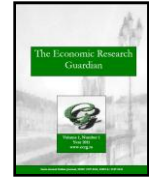


Table 2 - Non normalized values of $U_i(t)$ in the period 2003-2011

	IT	FR	GER	SP	UK
2003	4,16	3,97	4,24	4,95	4,58
2005	4,16	4,45	5,36	5,19	5,42
2006	5,65	4,24	5	5,57	4,68
2007	5,05	5,8	5	6,08	7,44
2008	7,63	5,27	8,14	6,57	6,11
2009	5,1	4,58	6,82	6,4	6,69
2010	4,63	4,04	7,17	5,49	5,12
2011	5,27	4,98	5,99	6,15	4,68

In Table 3 we report the correlation of the values of $U_i(t)$ in the 8 years of the couples of members of the club. A large correlation would imply that the two economies considered may experience the same evolution. This is the case of the Italian and Spanish economies. On the contrary, a small value of the correlation signals different evolutions, as in the economies of France and Germany.

Table3 - Correlation of the values of $U_i(t)$ between the members of the club in the 8 years

ital-fra	0,49132
Ita-Ger	0,650011
it-spa	0,744121
it-UK	0,230873
fra-ger	0,193899
fra-spa	0,708769
fra-UK	0,729618
ger-spa	0,669752
ger-uk	0,248491
spa-uk	0,594851

In Table 4 we report the standard deviation of the values of $U_i(t)$ in the 8 years for the 5 members of the club. Note that each value of the standard deviation is normalised to the value of $U_i(t)$ of the club in that year.

Figure 4 shows graphically the oscillations of the spreading of the club members. One may note a first relative maximum around 2006 followed by the maximum spreading of the club reached in 2010 and the relevant recovery in the following year.

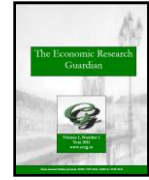


Table 4 - Normalised early spread (standard deviations) of the values of $U_i(t)$

2003	0,0919
2005	0,0826
2006	0,2031
2007	0,1737
2008	0,1708
2009	0,0752
2010	0,2313
2011	0,0639

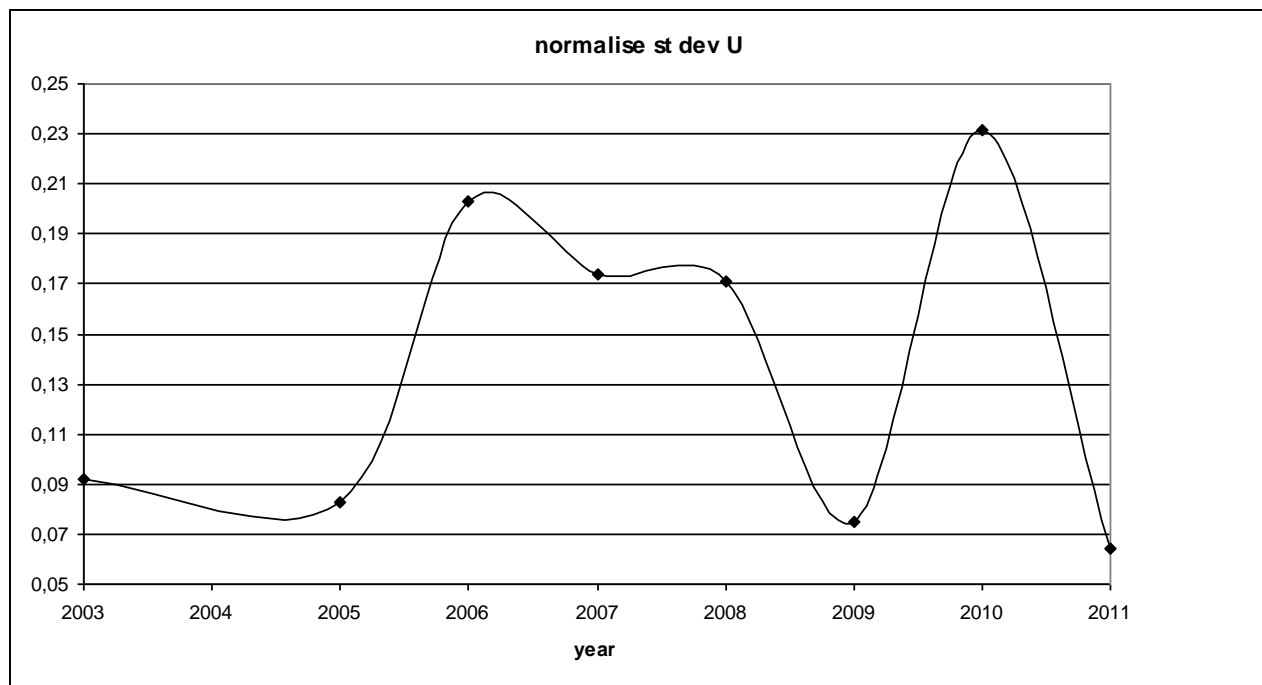
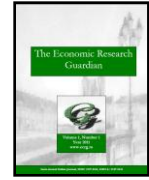


Figure 4 - Normalised yearly spreading

We note the large variation of the normalized spreading of the $U_i(t)$ of the club in the period 2006 – 2010 and a subsequent sudden return, nearly to the previous value. The disaggregation of the spreading, in relation to the behaviour of the various countries, may give a picture with sub groups of members of the club with tendentially homogeneous oscillations of their small internal spreading and dis-homogeneous to other sub groups. The reasons may be object of interesting researches.

They may be done by analysing the spreading of the various parameters (presented in Table 1), contributing to the curves of the individual spreading of the various members of the club, in every year considered. This will be the topic of the next section.



2.6. The weight of the parameters

The degree of homogeneity, stability or spreading of the members of the club estimated here is likely to be associated with the spreading of the parameters describing the single members of the club.

Table 5 - Normalized spreads of $U_i(t)$ and these of all 15 parameters

	2003	2005	2006	2007	2008	2009	2010	2011
norm s.d.U	0,091861	0,08256048	0,203136	0,173788	0,1708325	0,075253	0,231306	0,063862
sum s.d.par	0,289263	0,37870027	0,334067	0,318282	0,3466853	0,351313	0,322544	0,327118

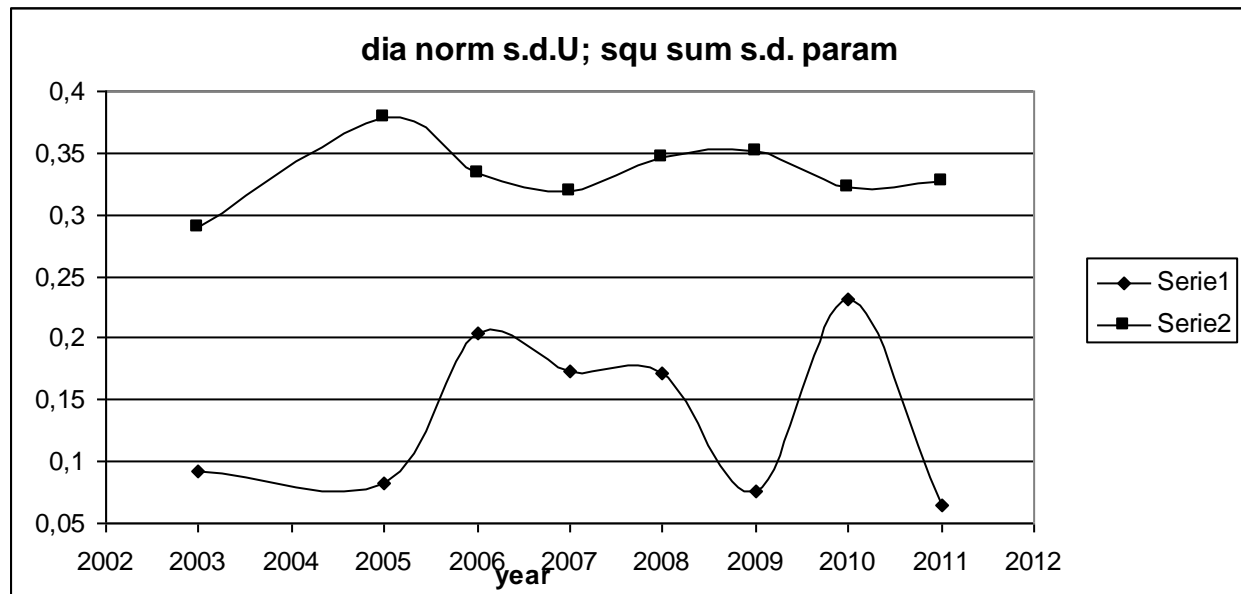
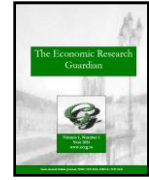


Figure 5 - Sum of the spreads (standard deviations) of all normalized parameters (diamonds) and normalized spread (normalized standard deviation) of $U_i(t)$ (squares)

We already mentioned the large variations of inhomogeneity occurred between the years 2007 and 2008 in Germany (8.14-5) and Italy (7.63-5.05) and in between the years 2006 and 2007 in the UK (7.44-4.68). The question about the parameters may then arise, which could be responsible for this behaviour. To respond, we begin the study of possible correlations between the measured quantities.

In Figure 5 the lack of correlation (- 0,182) between the sum of the yearly standard deviations of all the normalized parameters with the yearly normalized standard deviation of the values of $U_i(t)$ clearly emerges.



The question of a possible correlation with the single parameters remains open.

Table 6 shows the correlations between the standard deviation of the normalized values of the yearly $U_i(t)$ value of the club and the yearly standard deviations of all the parameters of Table 1.

Table 6 - Parameters correlation

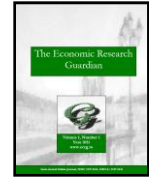
Parameter	Correlation
1	-0,32256
2	-0,31587
3	-0,46125
4	-0,36266
5	0,465773
6	-0,10199
7	0,219416
8	0,247192
9	-0,20267
10	0,262145
11	-0,06642
12	0,115932
13	-0,28095
14	0,42995
15	0,020263

Table 6 shows that the standard deviation of the normalized values of the yearly $U_i(t)$ of the club and the standard deviations of the parameters are not correlated. The lack of correlations may imply that the endogenous factors that determine each parameter in the general equilibrium or disequilibrium of the various economies, in the various years, have been influenced differently by some exogenous factors. The different behaviour of the considered parameters implies different elasticity to the impulses of the endogenous and exogenous variables, which may be captured by the memory formalism represented by the fractional derivatives of the considered parameters.

The lack of correlation between the spread of the parameters and the measure of the spread of $U_i(t)$ may not be surprising if the effect of each variable of the economy is delayed by the different parameters of the memory mechanism of each of the considered homogeneity parameters.

The exclusion of the existence of a direct effect of the spread of the single or all normalized parameters on the normalized spread of $U_i(t)$, implies that the direct use of the values of the parameters and of their spread is not viable for estimating the evolution of the spreading of the members of a club. Our approach is more reasonable and acceptable.

The method implied by the model structure suggests a numerical measure of the phenomena of homogeneity and stability, which is the first necessary step. But in this way, we have not made clear if



there is some dominant factor, observable within our homogeneity parameters, that generates the impulses leading to changes in the relevant variables affected, as for their elasticity, by different memory mechanism, with different delays for the various parameters¹⁹.

A first step in this identification may consist in observing which parameter has given the greater contribution to $U_i(t)$ spread, in the various years and if some parameter emerges with a systematically larger contribution. Obviously, one may argue that that parameter is the most elastic to the changes, and that, because of its greater variations, *ceteris paribus* about the interdependence factors shall exert a greater influence on the other parameters.

From the analysis, the contribution of the 15 parameters in the period 2003-2011 emerges that GDP growth gave the greatest contribution to $U_i(t)$. Obviously, GDP growth is the result of many variables that alter its trend upward or downward or to (quasi)stagnation. At the same time, it is also the most relevant variable that may influence the other homogeneity parameters considered.

Indeed, there are either important structural components of GDP growth dynamics as GDP per capita (causing fluctuations of the population too or important endogenous causal factors of GDP behaviour as saving/GDP, investment /GDP, labour product per hour and per person, VA agriculture and VA industry and public expenditure/GDP. The remaining homogeneity variables of the CLUB considered in our model - unemployed/GDP, government deficit/GDP, bond yield and inflation rate, balance of payment and balance of payment current - are exogenous variables generally, strongly and interdependent with GDP growth dynamics.

Therefore, we consider the GDP growth spreading in the 9 years and correlate it with the spreading of $U_i(t)$ in the same years, for the 5 members of the club.

In Figure 7, in order to assess the correlation between the spreading of the parameters and that of $U_i(t)$ in the 8 year considered, we show the data relative to the spreading of the average $U_i(t)$ and of GDP in the 8 years considered.

¹⁹ On the lack of resolution that does not allow a detailed analysis of the correlation function in order to see the important delay between investments and GDP per capita or in general in monetary policy (e.g. Caputo, 2005, 2009, and Caputo and Di Giorgio 2006).

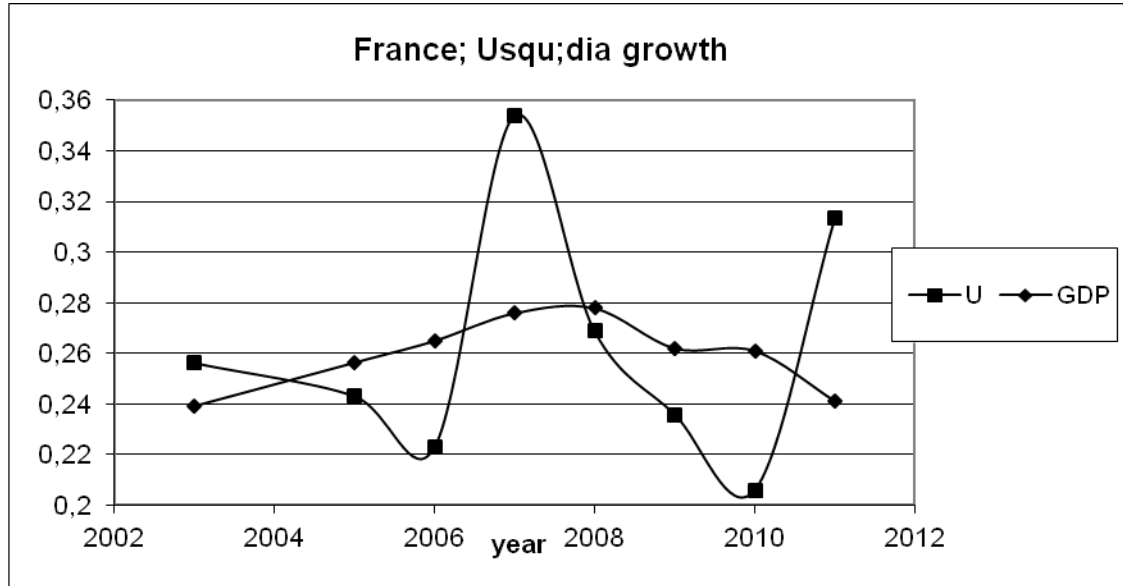
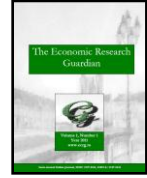


Figure 7.I - France. GDP spread and $U_i(t)$

The correlations between the sets of data in Figure 7 are negligible: 0.17 between parameters 1 and 2, 0,44 between $U_i(t)$ and parameter 1, and -0.37 between $U_i(t)$ and parameter 2.

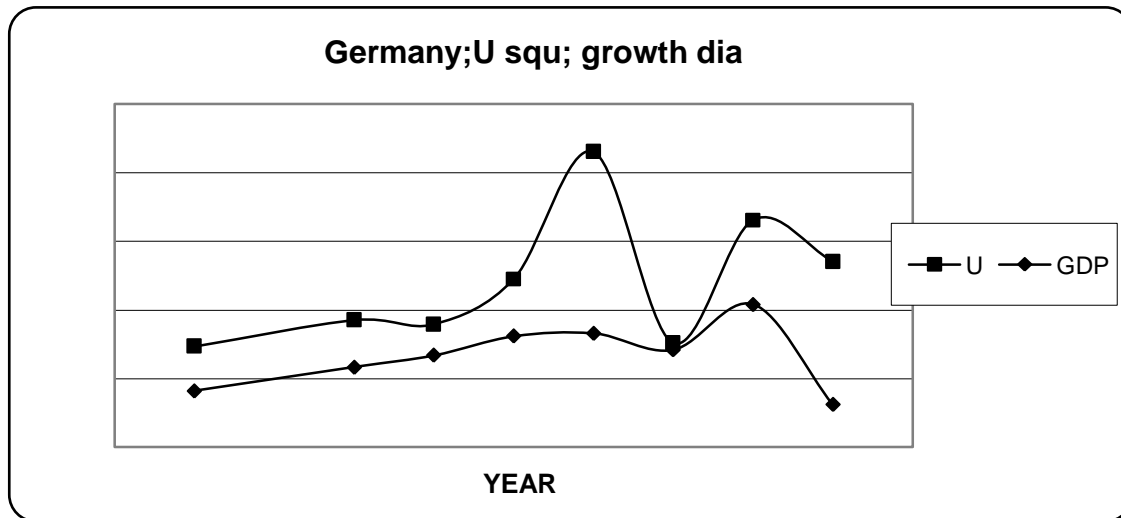


Figure 7.II - Germany. GDP spread and $U_i(t)$

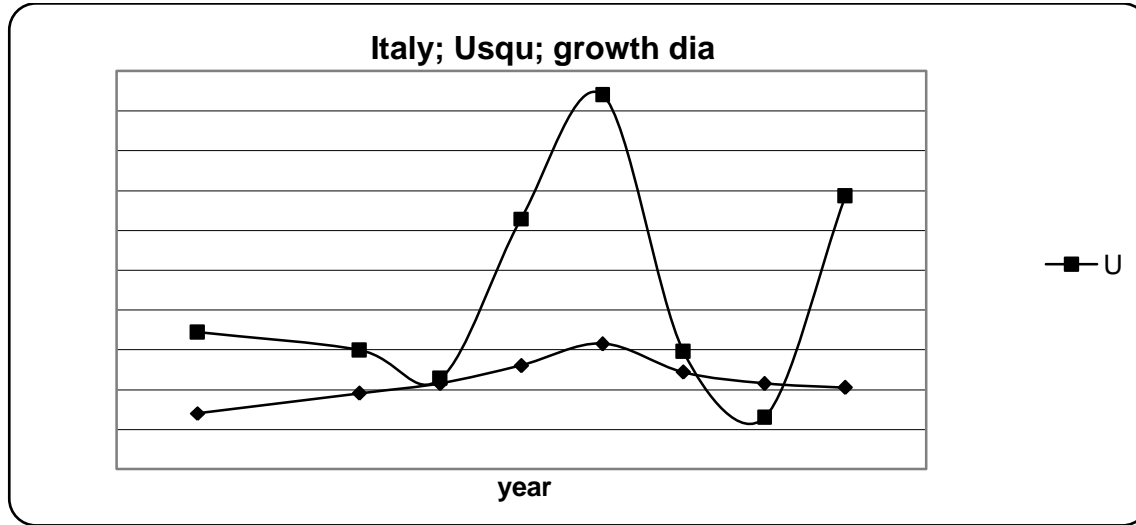
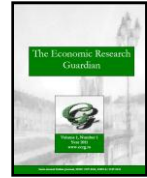


Figure 7.III - Italy. GDP spread and $U_i(t)$

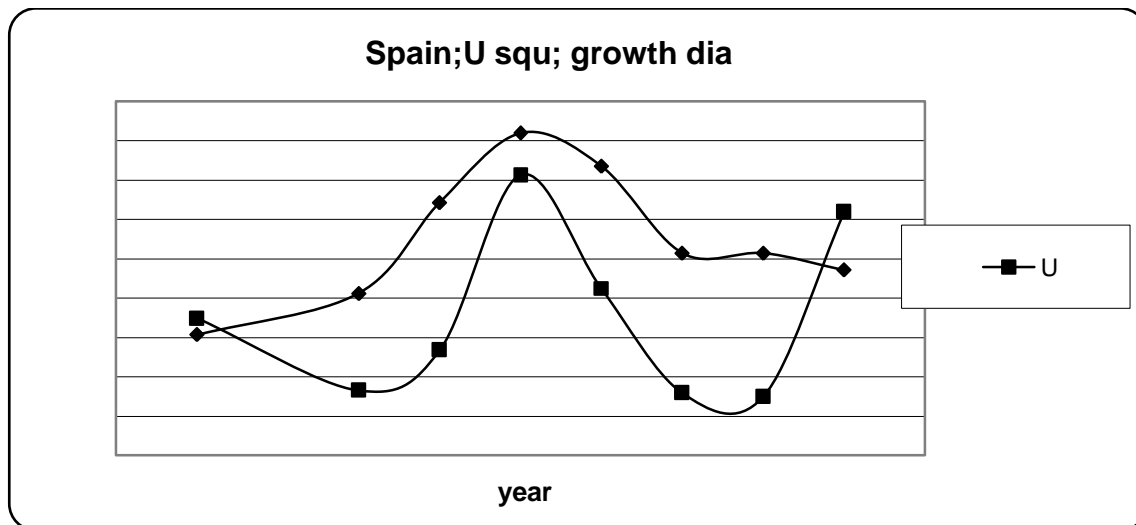


Figure 7. IV - Spain. GDP spread and $U_i(t)$

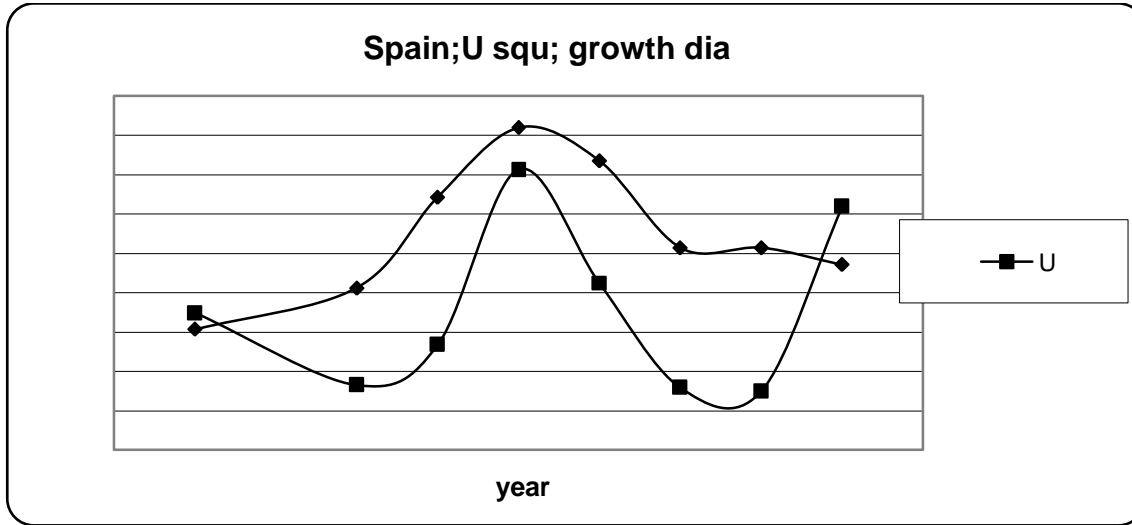
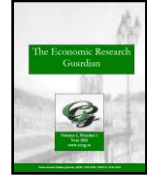


Figure 7 - United Kingdom. GDP spread and $U_i(t)$

It is clear from the Figures 7I, II, III, IV ,V, relating to the 5 countries considered, that the parameters are in different relation with the spread of $U_i(t)$.

We also test the three correlations between the spreading of the average $U_i(t)$ and of parameter 1, with large spreading and of parameter 2 with smaller spreading and between parameters 1 and 2 in the 8 years period.

Figure 8 shows the average values of $U_i(t)$ (diamonds), of parameter 1 (squares), parameter 2 (triangles).

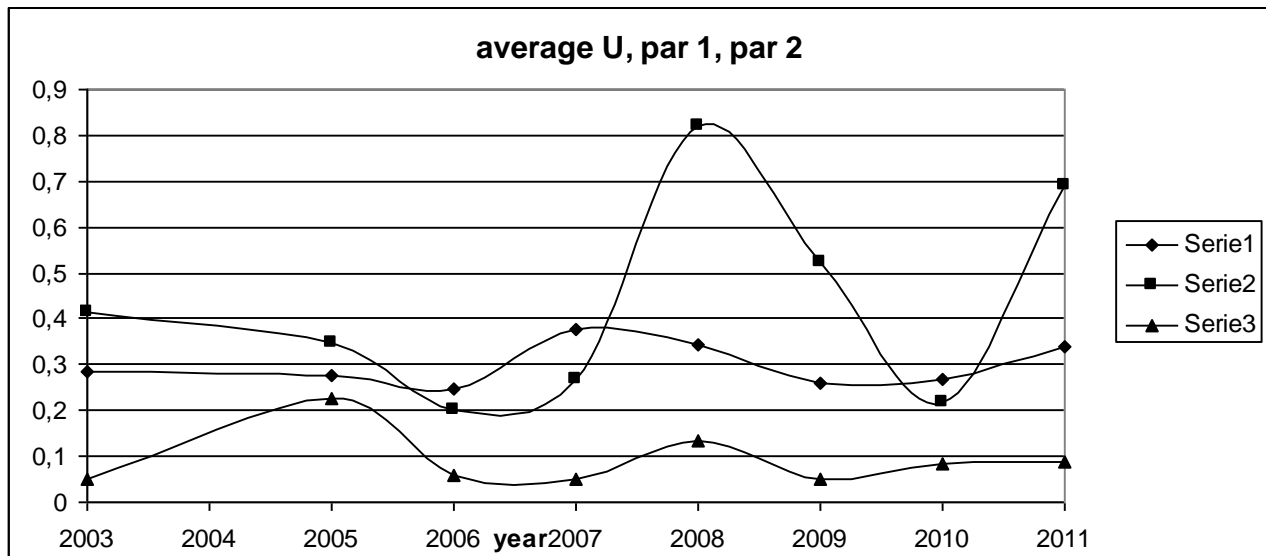
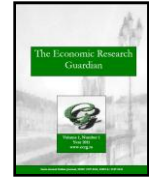


Figure 8 - Large and small spreading parameters and EU spread



The two correlations between $U_i(t)$ and GDP growth and GDP pro capita are negligible and opposite, i.e. respectively 0,44 ND -0,37. That between parameters 1 and 2 is particularly low, of 0,17.

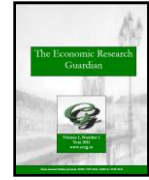
The low correlation between $U_i(t)$ and GDP growth parameter 1, which is generally responsible of the variation of the spread of $U_i(t)$, implies the relevance of the effect of the other 14 parameters, although their singular contribution would be smaller than that of parameter 1.

It remains to explain the extremely weak correlation between GDP growth and GDP pro capita and their opposite relation with $U_i(t)$, one positive and the other negative. The memory displays a much more different role as for the aggregate GDP growth dynamics and for the GDP per capita, because of the relevance of the migrant share of the population of the five EU main developed countries member of EU club and of the four member also of the EMU club. The memory of the migrant is smaller than that of the population resident there from a long time. Therefore, for them the exit implies a smaller loss of rents of the localization. Migrants may move easily, both from the five considered countries to their origin countries in EU and non EU countries, and among the five EU countries. The homogeneity of the club, as for the per capita population of each country member of the considered club, is higher than that of their aggregate GDP. On the other hand there cannot be a correlation between the EU aggregate spread and that of the GDP per capita because the mobility inside the five considered main develop countries increases the population of some of them and decreases the population of some other, while the mobility outside them may also differ.

On the other hand, a likely explanation for the greater variation of the spread of GDP growth parameter compared to that of any other homogeneity parameter of the five main developed countries, may be found in the policies of EU and EMU. The financial variables of the countries of the two EU and EMU clubs are deeply influenced by these policies and thus, in turn, these variables interact with the GDP of the considered countries in different ways.

It also seems clear that while before the boom and the subsequent crisis, convergence was emerging together with a small but not irrelevant GDP growth, the period of cyclical fluctuation has caused a large spread in GDP dynamics, influencing the other club stability parameters in various ways and intensity in the various countries. After the period of cyclical fluctuations and the return to convergence, the divergence seems to reappear, with a semi stagnation situation. It seems that the Government of the EU and of the EMU clubs with the policies adopted could allow the “invisible hand” of the market competition to generate convergence with (modest) growth. However, they have been unable to control the cycle and lead the main developed countries member of EU and of EMU clubs to divergence, through a policy of national consolidation not compensational, at the macro EU and EMU level, by adequate expansionary EU expansionary fiscal policies and by a timely non conventional expansionary monetary policy, under stable currency, i.e. a stable modest rate of price increase and a substantial global balance of payment equilibrium as for EMU. However, under the present policies or lack of policies, consolidation has been obtained so far by reducing growth to a semi stagnation stage with different rates in most of the countries considered.

Under the exchange rate channel, a monetary expansion depreciates the local currency and hence lowers the relative prices of domestic goods, which raises demand and supply of domestic goods and services. Moreover in this case there is an immediate and more direct effect on inflation through the



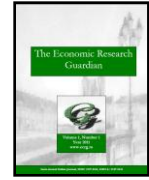
impact of the depreciation on prices of imports. Based on literature the magnitude and speed of these effects depend on the country's degree of openness, dependence on imported goods, and competitiveness. Also, the pass-through to prices can be asymmetric if prices are downward sticky, whereby inflation is slower to adjust downward following appreciation than upward following depreciation.

3. Concluding remarks

We have examined the viability of EU and EMU as clubs, from a public choice approach to the multilevel structure of governments, considering the five main EU-developed countries, four of which they are also members of EMU. Clubs that favour convergence cum GDP growth of the member countries, must have a tendency to homogeneity. Therefore, they need to adopt policies conforming to two not necessarily consonant: homogeneity and GDP growth. In the public-choice theory, the option of exit eases the homogeneity of the club. On the other hand, in order to be stable, the monetary union and the union of governments with different currencies must be characterised by unlikelihood of exit, at least for their main countries. Thus, a reinforced governance is needed to assure long run stability. We have identified 15 convergence parameters, selected by considering the factors of convergence in the main models of growth and the rules of the EU and EMU constitutions. We have, subsequently adopted a model to measure the convergence spreads among the 15 parameters. We have also assumed that in order to explain the convergence-divergence trends among the economic and financial variables of the model, in an appropriate perspective for the evolutionary behaviour of human communities, a memory formalism expressed by fractional derivative formulations may be useful. We then tested the convergence in the EU and EMU in the 2001-2011 period, in which, as it is well known, an economic deep fluctuation, mainly of financial origin, took place.

It clearly results that convergence, or at least stability cum moderate GDP growth, was taking place before the cyclical fluctuation. Under it, a new divergence wave in the sum of the 15 parameters emerged among the five main countries in EU, after a limited convergence period. Divergence did reappear, however with stagnation. The analysis of the behaviour of each of the 15 parameters revealed that the largest spreading belongs to GDP growth and that the smallest belongs to GDP per capita. The reasons why GDP dynamics is the parameter with the highest instability may be that this is the most important factor affecting the other variables and affected by them, even if it is in different ways. GDP growth rate is the parameter most influenced by the policies adopted to control the cycle. The lack of correlation of GDP growth dynamics and the other 14 variables may be explained with the different memory formalism and, in the case of GDP per capita, by the low impact of memory on the migration waves.

After the period of cyclical instability and the convergence trend, divergence seems to reappear, with a semi stagnation situation. It seems that the Government of the EU and of the EMU clubs, with the policies adopted so far, are unable to control the cycle and to lead the main countries of the two clubs to regain convergence to a homogenous-stable growth path.



In order to assure stability cum growth of the two clubs, one needs EU and EMU monetary and fiscal policy tools coherent with their models, which are already institutionally available and not employed or employed with delays. However, additional EU common rules are needed to complete the unique market, particularly as for the labor contracts. Further integration would face the same issues, in a less free situation.

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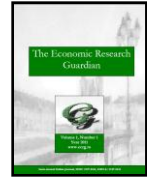
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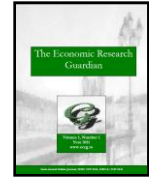
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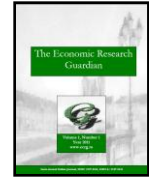
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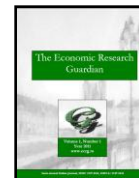
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